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Rolf Lagerborg

PROFESSOR ROLF LAGERBORG, who died on 26th October 1959, was born in 1874 in Finland, where he spent the whole of his academic career, first as a lecturer at Helsingfors (1909) and then as professor at the Swedish university of Åbo (1929).

Lagerborg was the author of a considerable number of works, which ranged over an extremely wide field. Ethics was one of his great interests. As a pupil of his fellow-countryman, Edv. Westermarck, and a follower of the French sociological school, he published a number of works in which he applied psychological and sociological criteria to the analysis of moral phenomena. Alongside Westermarck's internationally recognised book, "Ethical Relativity", Lagerborg's "Moral i vardande" ("Morals in the making" 1937) is one of the most important Scandinavian contributions to this tradition. Lagerborg's concept of morality differed from Westermarck's, however, in that he laid more stress on sociological aspects. To the same sphere of interest belongs "Om sanningen i religionen och fromhetslivet" ("On truth in religion and pious life" 1934), which also shows considerable dependence on the ideas of Durkheim.

Several of Lagerborg's other published works were devoted to psychological questions, including the early "Das Gefühlsproblem" (his doctoral thesis, 1905) and last of all the little booklet "Reaktionspsykologi" ("Psychology as a Theory of Stimulus and Reaction" 1943), which, with its insistence on a purely scientific view of psychological phenomena, he regarded as the essence of his teaching. This little booklet is a masterpiece of clear exposition

and sustained argument. His belief in a modified form of behaviourism was combined with a positive interest in psychoanalysis, and he was one of the first in Scandinavia to succeed in arousing public interest in this branch of psychology.

His wide reading and scholarship also extended to Greek and Roman antiquity and his account of "Die Platonische Liebe" (published first in Swedish, 1915) has long been a standard work. His numerous essays, of which some of the best were collected in his book "Invita Minerva" (1918), have had an important part to play in Swedish literature.

Lagerborg was an original thinker and led a full and busy life. As a liberal and a rationalist, he had considerable admiration for the "enlightenment" of the 18th century. He thrived on controversy and was an ardent enemy of religious and national prejudices – and this meant, of course, that he himself was the subject of controversy. For those who shared his view of life and the ideals to which he devoted himself, he was one of the most stimulating authors in the whole of Scandinavia. At times naive, but on the whole clear-sighted and significant; occasionally egocentric, but even so a man with great aims; always fearless and yet full of charm. His great autobiography (1942, 1945) is an invaluable record of cultural life in Finland during the first half of this century and at the same time a fascinating portrait of Lagerborg as a man, a happy warrior, but full of self-searching.

His death leaves a gap in Scandinavian culture.

Ingemar Hedenius

Some problems concerning disposition concepts

by

JAN BERG

(University of Stockholm)

§ 1. *Introduction*

By an (*explicit*) *definition* of the predicate P we mean an expression of the form:

$$Dx \equiv . \quad - \quad - \quad - \quad x \quad - \quad - \quad - ,$$

where (i) P is 'D' (ii) the expression ' $- \quad - \quad - \quad x \quad - \quad - \quad -$ ' contains only ' x ' as a free variable, and (iii) P is not included in ' $- \quad - \quad - \quad x \quad - \quad - \quad -$ '. The expression A is a *definition* of the predicate P in the language L if, and only if, (i) A is an explicit definition of P , and (ii) A is true in L . (A may not be analytic in L .)

In this paper, all definitions are considered as sentences within a language L and not as translation rules between two languages. The subsequent discussion does not concern purely conventional definitions, but only such explicit definitions that form a reconstruction of linguistic usage. Our task will be to analyse definitions that are explicative rather than descriptive in nature. However, some traits of linguistic usage considered as particularly important will be taken into account throughout.

By the expression '*disposition*' we mean the property of an object to react in a certain manner R when submitted to the test Q . If D is a disposition with respect to Q and R , we have, according to common linguistic usage, a right to state:

$$Dx \equiv . \text{ If } Qx \text{ were, then } Rx \text{ would be.}$$

The test Q and the reaction R are said to be *correlated* to the disposition D if, and only if, D is a disposition with respect to Q and R . If D is such a disposition, then:

$$Dx \equiv . Qx \supset Rx,$$

obviously does not generally hold.

§ 2. Two problems of definition: (I) and (II)

The question of defining disposition predicates is of importance in the construction of empiricist languages, that is languages where the meaning of the sentences can be specified in terms of a basic observational vocabulary by means of logical constants.

The general problem of defining disposition predicates can be looked at from two points of view. On the one hand, it can be the problem to find, in a language L , a definition of a special disposition predicate P , i.e.,

- (I) to find, for a disposition predicate P , a true sentence in L of the form:

$$Dx \equiv . \quad \text{---} x \text{---},$$

where P is 'D', and the expression '--- x ---' contains only ' x ' as a free variable and does not contain P .

On the other hand, the problem can be to find definitions of disposition predicates for all languages that are sufficiently expressive in regard to logic, i.e.,

- (II) to find a *scheme of definition*:

$$Dx \equiv . \quad \text{---} x \text{---} Q \text{---} R \text{---},$$

which – for all languages L that are sufficiently expressive in point of logic and that contain a disposition predicate P_1 , a predicate P_2 , denoting the test correlated to the denotation of P_1 , and a predicate P_3 , denoting the correlated reaction – turns into a *definition* of P_1 in L , when P_1 is inserted for 'D', P_2 is inserted for 'Q', and P_3 is inserted for 'R'.

In case (I), evidently, the definiens may contain other predicates of L besides 'Q' and 'R'. For instance, the sentential function of the definiens can be made up of one single predicate of L with the free variable ' x ':

$$Dx \equiv . Fx.$$

Then, if D is a disposition with respect to Q and R , and if L is logistically formalized, it should often happen that the following sentence would be true in L (with ' $\forall y$ ' abbreviating 'for all y '):

$$\exists y(Fy \& Qy) \& \forall y((Fy \& Qy) \supset Ry).$$

Also, the definiens could be composed of a disjunction of predicates of L :

$$Dx \equiv .F_1x \vee F_2x \vee \dots \vee F_nx,$$

or a sentential function of some other form.

In case (II), it follows from the explanation of the term 'definition' in § 1 that ' Q ' and ' R ' are the only free variables in the right member of the scheme taking predicates as their values. However, the right member may contain bound predicate variables or predicate constants. In the sequel, when not otherwise indicated, we presume the logic underlying L to be *extensional*. Hence, there is no need of distinguishing between classes and properties.

§ 3. Kaila's attempt at solving problem (II)

Eino Kaila has developed an idea of definition with the intent of answering problem (II).¹ Kaila assumed that if D is a disposition with respect to Q and R , then x has D in those cases when $Qx \& Rx$, and further when $\sim Qx$ and there is some class F such that Fx and such that general implication holds from ' $Fy \& Qy$ ' to ' Ry '. Lest it be possible to construct general implications with a vacuous antecedent and thereby attribute dispositional properties in a way that conflicts with linguistic usage, Kaila added that there must be a y such that $Fy \& Qy$. His scheme of definition is as follows:

$$(Df_k) \quad Dx \equiv . \exists F(Fx \& \exists y(Fy \& Qy) \& \forall y((Fy \& Qy) \supset Ry))$$

The variable ' F ' is intended to range over "natural" or "inductive" classes.² These classes shall admit of an inductive generaliza-

¹ Kaila 1939, pp. 239-240. (Names followed by a date refer to the bibliography.)

² Kaila 1942, p. 33.

tion from known to unknown cases in the sense that if P is a property which occurs with all known elements of the class K , then it is a justified assumption that P also occurs with all hitherto unknown elements of K .

Apart from the problem of inductive classes, however, there remains a crucial objection against (Df_k) . Suppose that

$$\exists x(Dx).$$

Hence, by (Df_k) :

$$(1) \quad \exists y(Qy \ \& \ Ry).$$

Assume further:

$$(2) \quad \sim Qx.$$

Then, by (1) and (2):

$$(3) \quad (Qx \supset Rx) \ \& \ \exists y((Qy \supset Ry) \ \& \ Qy) \ \& \ \forall y(((Qy \supset Ry) \ \& \ Qy) \supset Ry).$$

For many dispositions D , the class $\hat{x}(Qx \supset Rx)$ will belong to the range of 'F'. We assume that our D is of this sort.³ Then, from (3), we can infer:

$$\exists F(Fx \ \& \ \exists y(Fy \ \& \ Qy) \ \& \ \forall y((Fy \ \& \ Qy) \supset Ry)),$$

and so, by (Df_k) :

$$Dx.$$

Thus, we see that if something has a D of the not uncommon sort just mentioned, (Df_k) entails:

$$\forall x(\sim Qx \supset Dx),$$

i.e., everything not tested with respect to D has D , which is contrary to linguistic usage.

§ 4. Storer's analogous attempt at solving (II)

Storer 1951 propounds a scheme of definition for disposition predicates which is akin to (Df_k) . With the present terminology (enlarged by the time variable 't'), Storer's scheme would look:

³ This device originates from Wedberg 1942, p. 44. Cf. also Wedberg 1944, p. 237.

(Df_s) $Dx \equiv \exists t(Qx, t \ \& \ Rx, t) \vee \exists F(Fx \ \& \ \exists y(Fy \ \& \ \exists t(Qy, t \ \& \ Ry, t))) \ \& \ \forall y((Fy \ \& \ \exists t(Qy, t)) \supset \exists t(Ry, t))$.

This scheme contains unnecessary expressions. Storer's own restriction of the range of 'F' (cf. below) admits $\hat{x}\exists t(Qx, t \ \& \ Rx, t)$ as a value of 'F', and hence the second term of the definiens follows from the first one, which therefore is superfluous. Further, it seems uncertain whether the concluding implication of the definiens corresponds to linguistic usage. If an object x has a disposition, then we should say that it holds of *any time* t that if x is tested at t , then it reacts at t .

If (Df_k) and (Df_s) are considered without additions as to the range of 'F', we can show that (Df_s) is weaker than (Df_k). Upon introduction of 't' into (Df_k), the following scheme of definition emerges:

(Df_{k'}) $Dx \equiv \exists F(Fx \ \& \ \exists y\exists t(Fy \ \& \ Qy, t) \ \& \ \forall y\forall t((Fy \ \& \ \& \ Qy, t) \supset Ry, t))$.

Now it can readily be shown that the definiens of (Df_s) follows from the definiens of (Df_{k'}).

According to Storer, the variable 'F' ranges over classes which are (i) undefined or (ii) defined without explicit reference to individuals.⁴ Even with this restriction inserted into the definiens, (Df_s) leads to undesirable results in many cases. For example, if the language L contains (Df_s), and if 'Q' and 'R' are observation predicates undefined in L , then L contains the expression ' $\hat{x}(\exists t(Qx, t) \supset \exists t(Rx, t))$ ', which defines a class K without containing any undefined names of individuals. Now, if K is not defined in L in any other way that explicitly refers to individuals, K will belong to the range of 'F'. In these cases we can show, by an argument similar to the one about (Df_k), that if $\exists x(Dx)$, then (Df_s) entails:

$$\forall x(\sim \exists t(Qx, t) \supset Dx).$$

⁴ Storer 1951, p. 137 n.

§ 5. *Reichenbach's analysis of nomological sentences*

That D is a disposition with respect to Q and R is usually expressed in ordinary language by means of verbs in the subjunctive mood:

$Dx \equiv$. If Qx were, then Rx would be.

Hempel⁵ has outlined a program for the solution of problem (II) where the interest is directed towards subjunctive sentential functions. If the exact meaning of those functions could be determined, it would be possible to construct a scheme of definition for disposition predicates.

An attempt to interpret a certain class of subjunctive sentential functions has been made by Reichenbach.⁶ His analysis is based upon the concept of nomological sentence, which is intended to embrace both logical and empirical laws. A sentence is nomological if it follows from an original nomological sentence, and a necessary condition for a sentence to be nomological and original is that it contains only predicates which are universal, in the sense that they are defined without the aid of individual names.

Now it is plausible to believe that Reichenbach's analysis of nomological sentences could be of importance for the solution of problem (II). However, this analysis does not lead us further, as will be shown in the next section.

§ 6. *Two kinds of subjunctive sentential functions*

To the general form:

(a) If ... x ... were, then --- x --- would be,
there belong two kinds of subjunctive sentential functions. The first kind consists of (i) those instances of (a) in which the variable ' x ' occurs essentially. A variable V occurs essentially in the sentential function S if, and only if, replacement of V in S by names of individuals from the range of V turns S sometimes into a true sentence and sometimes into a false sentence. The second

⁵ Hempel 1950, pp. 54 f; Hempel 1952, p. 25.

⁶ Reichenbach 1947, ch. 8. Cf. also the posthumous Reichenbach 1954.

kind of subjunctive functions comprises (ii) those instance of (α) in which the variable ' x ' occurs *vacuously*, i.e., not essentially.

In case (i), the universe of discourse is divided into the class of all things satisfying (α) and the class of all things that do not satisfy (α). Obviously, the subjunctive functions needed in the analysis of dispositional concepts are of kind (i).

Sentential functions of kind (ii), on the contrary, are either universally true or empty. Reichenbach's analysis concerns all subjunctive hypotheses which are stated with a claim of being "natural laws" and which, hence, are of kind (ii). Now, according to Reichenbach, if A is a subjunctive sentential function, where ' x ' is the only free individual variable, and if $\forall x(A)$ is stated with the claim of being a "natural law", then $\forall x(A)$ is equivalent to a sentence of the metalanguage expressing that the general implication from the antecedent of A to the consequent of A is nomological.

The question is whether subjunctive functions of kind (ii) may be used in the analysis of those of kind (i). Bearing in mind the discussion of § 3, we could perhaps analyse the sentential function 'If Qx were, then Rx would be', which contains essential occurrences of ' x ' and expresses that x has a certain disposition, by saying that there is a class F such that Fx and such that the function 'If Fy and Qy were, then Ry would be', where ' y ' occurs vacuously, is true for all ' y '.⁷ In Reichenbach's terminology, such a solution to problem (II) could be formulated:

(Df_r) $Dx \equiv \exists F(Fx \ \& \ ' \forall y((Fy \ \& \ Qy) \supset Ry) \text{ is nomological})$.

That (Df_r) will not do as a scheme of definition is easily seen, since the above-mentioned objection to (Df_k) can, with a slight modification, be applied here also. If ' Q ' and ' R ' are predicates that may appear in nomological sentences, the universal property $\hat{x}(Qx \supset Rx)$ could be taken as a value of ' F '. Since the class of

⁷ A proposal in this direction is stated in Diggs 1952, p. 518, where subjunctive sentential functions are analysed in the manner of (Df_k). Note also the relationship between our interpretation of (i) in terms of the nomological implication (ii) and Myhill's interpretation of ' $A \supset B$ ' in terms of strict implication in Myhill 1953.

logical theorems is included in the class of nomological statements, the sentence:

$$\forall y(((Qy \supset Ry) \ \& \ Qy) \supset Ry)$$

is nomological, and we once again get the consequence:

$$\forall x(\sim Qx \supset Dx).$$

This shows that there is a large class K of subjunctive sentential functions such that even if we succeed in giving an interpretation of all elements of K, this would not, by methods of definition available so far, help us to a satisfactory scheme of definition for disposition predicates.

§ 7. *A non-extensional approach to problem (II)*

A try at amending (Df_r) by a stipulation that excludes logical theorems from the class of nomological statements will be of no use.* This appears from the following analysis of a non-extensional treatment of problem (II) given in Burks 1955. There a causal modality (which we will denote by 'N₁') is postulated in a language, and then other modalities are defined in that language by means of N₁ and logical necessity (which will be denoted by 'N₀'). Via the causal necessity N₂:

$$N_2A \equiv N_1A \ \& \ \sim N_0A,$$

Burks reaches the concept of non-paradoxical causal implication: $npc((A_1 \ \& \ A_2 \ \& \ \dots \ \& \ A_p), B)$. Like strict implication, this concept is a two-place modality. We will reduce it here to the one-place causal modality N₃ and material implication:

$$npc((A_1 \ \& \ A_2 \ \& \ \dots \ \& \ A_p), B) \equiv N_3((A_1 \ \& \ A_2 \ \& \ \dots \ \& \ A_p) \supset B).$$

If we include quantifiers of first order, N₃ may be defined in connection with an important special form of implication sentences as follows:

* (Added in proof.) For an unknown reason, Arthur Pap denies this in his otherwise excellent article *Disposition Concepts and Extensional Logic*, *Minnesota Studies in the Philosophy of Science*, vol. 2 (1958), pp. 212-213.

$$N_3 \forall x ((A_1x \& A_2x \& \dots \& A_px) \supset Bx) \equiv .$$

$$\equiv . N_2 \forall x ((A_1x \& A_2x \& \dots \& A_px) \supset Bx) \&$$

$$\& \sim N_2 \forall x (J_1 \vee J_2 \vee \dots \vee J_q), (q=1, 2, \dots, p),$$

where J_i ($i=1, 2, \dots, q$) is one of ' $\sim A_1x$ ', ' $\sim A_2x$ ', \dots , ' $\sim A_px$ ', ' Bx '.

The second conjunct of the definiens is to rule out causally superfluous conditions in the antecedent and to guard against the paradoxes of causal implication, arising when the antecedent is physically impossible or the consequent is physically necessary.⁸

Burks's scheme of definition for disposition predicates can be set up as follows:

(Df_b) $Dx \equiv . \exists F(Fx \& N_3 \forall y ((Fy \& Qy) \supset Ry)).$

Here the range of 'F' is restricted to not directly observable, "enduring" properties, characteristic of a thing over long periods of time and relatively difficult to lose or to acquire.⁹

But if we take the "enduring" property $\hat{x}(Qx \supset Rx)$ as a value of 'F', then from:

$$Rx,$$

we can infer:

$$\sim Dx,$$

since by the definition of N_3 :

$$N_3 \forall y (((Qy \supset Ry) \& Qy) \supset Ry)$$

is false. Then (Df_b) entails that every x which shows a reaction correlated to a disposition D is lacking D , which certainly is unidiomatic.

§ 8. *A weaker problem: (III)*

We have seen that serious obstacles are connected with the problem of introducing disposition predicates in a general way

⁸ On redundancy, see Burks 1955, p. 180, n. 13. On the paradoxes of causal implication, see Burks 1951, p. 377.

⁹ Burks 1955, pp. 181-182; p. 188, n. 22.

in formalized languages by means of explicit definitions. Even a solution to the formidable difficulty¹⁰ of defining the universality or endurability of the property *F*, appearing in the schemes of definition discussed, would be insufficient for our purpose.

Instead of problem (II), now, we face the following problem:

- (III) We want a *sentential scheme* *S* such that 'D', 'Q', and 'R' are the only free predicate variables in *S*, and such that – for all languages *L* that are sufficiently expressive in regard to logic and that contain a disposition predicate *P*₁, a predicate *P*₂, denoting the experiment correlated to the denotation of *P*₁, and a predicate *P*₃, denoting the correlated reaction – it holds that if we insert *P*₁ for 'D', *P*₂ for 'Q', and *P*₃ for 'R', we get a *true expression in L* which assigns a *method for deciding* in at least one case whether *x* has or does not have the denotation of *P*₁. (It is not necessary that *S* always turns into a *definition of P*₁ *in L*.)

§ 9. Carnap's reduction method

In his theory of reduction sentences, Carnap¹¹ gave an answer to problem (III). For the sake of simplicity, only the part of the theory that considers so-called bilateral reduction sentences will be touched upon here. We define *A* as a (*bilateral*) *reduction sentence* for the predicate *P* in the language *L* if, and only if, (i) '∃*x*(*Qx*)' is a true expression in *L*, and (ii) *A* is a true expression in *L* of the form:

$$(R_b) \quad \forall x(Qx \supset (Dx \equiv Rx)),$$

where *P* is 'D'.

(*R*_b) demands only a vocabulary of the same richness of expression as the lower predicate calculus, while (*Df*_k) of § 3 re-

¹⁰ Cf. Goodman 1946; Goodman 1947, part III; Hempel & Oppenheim 1948, § 6; Burks 1951, section 2. 4.

¹¹ Carnap 1936, vol. 3, pp. 440 f.

quires a vocabulary comparable in strength with that of the predicate calculus of second order.

(R_b) is equivalent to the conjunction of (R_1) and (R_2) :

$$\begin{array}{ll} (R_1) & \forall x[(Qx \ \& \ Rx) \supset Dx], \\ (R_2) & \forall x[(Qx \ \& \ \sim Rx) \supset \sim Dx]. \end{array}$$

If we imagine (Df_k) with an addition in the right member:

$$(1) \quad Dx \equiv. \exists F[... F ... \ \& \ Fx \ \& \ \exists y(Fy \ \& \ Qy) \ \& \ \forall y((Fy \ \& \ Qy) \supset Ry)],$$

and if we imagine us having a formulation of ' $... F ...$ ' that waives the objection mentioned earlier (which would probably agree with Kaila's intentions), then (R_1) does not follow from (1), since ' $Qx \ \& \ Rx$ ' does not imply the right member of (1). Carnap and Kaila do not seem, then, to have had the same intuitive conception of dispositions.

§ 10. *An objection to Carnap's method*

But immediately we have to face a new problem that threatens the very foundations of Carnap's theory.¹² Since dispositions can be formed with any Q and R , (R_1) without restrictions could lead to strange results. Suppose we have a dispositional property Explosive of the following nature:

Explosive $x \equiv$. If somebody would knock on x , then x would burst to pieces.

Further, suppose that I knock on this table, and that some time later the table happens to burst. We should scarcely regard this as a sufficient condition for the table's being explosive. But if somebody knocks on the top of a shell, it will burst, and nobody would consider this as a matter of chance.

It may be that reduction sentences of the form (R_b) can be constructed only for those x 's where the coincidence $Qx \ \& \ Rx$ is

¹² A similar argument can be applied to Goodman's method of answering problem (II) for certain restricted disposition predicates. Cf. Goodman 1955, pp. 46 f, 50.

conformable to law. If this be so, the trouble is to eliminate the random coincidences. It is not sufficient to state that there must be more than one x such that $Qx \ \& \ Rx$, for as we have seen there may exist Q 's and R 's such that there are both objects x with random coincidences and objects x with non-random coincidences of $Qx \ \& \ Rx$. One way out is a demand in the antecedent of (R_1) for the existence of some intrinsic property which x shares with other things, and which is connected with $Qx \supset Rx$ according to a law. However, to elaborate and make precise this idea is as difficult as to solve problem (II), and if that were possible, there would be no need of bothering about the weaker problem (III).¹³

§ 11. *Dispositions as theoretical constructs*

In weakening our problem from (II) to (III), we have given up the ambition of introducing disposition terms by explicit definition and thereby making them eliminable. To introduce a term by reduction is a kind of axiomatic method.¹⁴ A new primitive is entered, and postulates like (R_1) and (R_2) are laid down for it.

If we carry this idea through to its conclusion, the trouble with random coincidences in (R_b) can be evaded. The disposition predicates formerly considered as terms in an observation language extended by reduction methods may be reconstructed as terms in a »theoretical» language, where predicates are introduced by postulates and only certain statements containing them are connected with statements about observables.¹⁵ Then a disposition D is theoretically combined with a range of other concepts, and the

¹³ Another way, eliminating many random coincidences, is the stipulation that x must be isolated when the experiment is done, i.e., during the experiment the immediate extrinsic conditions for x have to be unchanged from the application of Q . But a requirement of isolation does not rule out random coincidences that depend on intrinsic processes.

¹⁴ Cp. Goodman 1955, p. 60, n. 11.

¹⁵ See Carnap 1956, where some of the opinions held in *Testability and Meaning* are abandoned. Carnap does not deny, however, the possibility of introducing concepts into an observation language by reduction sentences.

positive or negative result of the test procedure based on the correlated Q and R should not always be taken as conclusive evidence for the presence or absence of D. Consequently, (R_1) and (R_2) will not be included in a language of this kind.

By thus treating disposition concepts on a par with theoretical constructs in science, the cluster of problems analysed above is dissolved.

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Digby and Berkeley on Notions

by

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It is well known that Berkeley distinguishes between ideas and notions; but it is also generally acknowledged that this distinction raises rather than solves a number of difficult questions. What exactly does Berkeley mean by 'notion'? What kind of knowledge is notional knowledge as compared with so-called ideal knowledge? These are two of the questions Berkeley seems to leave unanswered. Some parts of his philosophy stand on their own feet or very nearly so. His immaterialism is one such part. Yet, Berkeley did not merely intend to state a conclusive argument against the existence of material substances. He outlined what purports to be a philosophical system. In order to understand this system, one must consider and evaluate his view on notions.¹

In his analysis of the origins of Berkeley's philosophy Johnston suggested that Berkeley took the term 'notion' in its special sense from John Sergeant.² Certain passages in Berkeley, he pointed out, closely resemble comparable paragraphs in Sergeant.³ He

¹ A recent attempt to interpret Berkeley's view on notions is Sidney C. Rome's. Cf. "Berkeley's conceptualism", *Philosophical Review*, 1946, 55, pp. 680-86.

² Cf. G. A. Johnston, *The development of Berkeley's philosophy*, London 1923, p. 162.

³ *Ibid.*, p. 163-65. For Sergeant's philosophy see John Sergeant, *Sure-footing in Christianity or, rational discourse on the rules of faith*, London 1665. *Non ultra: or a letter to a learned Cartesian etc.*, London 1698. *Solid Philosophy, asserted against the fancies of the ideists etc.*, London 1697. I am indebted to the librarian of St. John's College, Cambridge, England, for a microfilm copy of this last book.

concludes, however, that there is no satisfactory answer to the question what notions are either in Sergeant or in Berkeley.⁴

Johnston's analysis, I submit, suffers from at least two shortcomings. He neither sees that Sergeant's view, though admittedly not too clearly stated in the paragraphs he quotes, is yet quite definite and intelligible. Nor does he seem to realize that this very view had been advanced long before Sergeant wrote his attack against the "ideists". In fact, Sergeant took it from his good friend Sir Kenelm Digby with whom it had originated almost fifty years before Sergeant defended it.⁵ This historical connection invites structural comparison between Digby's and Berkeley's metaphysics of notions.⁶ In the first section I shall therefore explain Digby's view.⁷ Its root, we shall see, is the denial of the existence of relations. This will be my cue for the second section, which will deal with Berkeley.

I

Digby's opponents are the later-day students of Aristotle who "introduced a model of ignorance out of his words".⁸ He himself

⁴ Johnston, *op. cit.*, p. 167.

⁵ Cf. Sir Kenelm Digby, *Two treatises, in one of which the nature of bodies; in the other, the nature of mans soul is looked into: in way of discovery of the immortality of reasonable souls*, London 1665. The second treatise has a special title page with the publication date 1645 and separate pagination. 'Notion' was also used in a technical sense by a contemporary of Digby's. Cf. Herbert of Cherbury, *De Veritate*, London 1656. Sergeant presumably met Digby through Thomas White. Cf. R. T. Petersson, *Sir Kenelm Digby: The ornament of England*, Cambridge 1956. Rémusat believes that Digby saw Sergeant in his house in Covent Garden. Cf. Charles de Rémusat, *Histoire de la philosophie en Angleterre depuis Bacon jusqu'à Locke*, Paris 1875, vol. 1, p. 300.

⁶ It is fairly certain that Berkeley was not acquainted with Digby's philosophy. However, there is a reference to Digby's book in the list of Berkeley's library. Cf. René Maheu, "Le catalogue de la bibliothèque de Berkeley", *Revue d'Histoire de la Philosophie*, 1929, 3, pp. 180-99.

⁷ I only know of two philosophical discussions of Digby's view, one by Cassirer, the other by Lasswitz. Cf. E. Cassirer, *Das Erkenntnisproblem*, Berlin 1922, vol. 2, pp. 207-15. K. Lasswitz, *Geschichte der Atomistik*, Hamburg and Leipzig 1890, vol. 2, pp. 188-207. Bréhier and Überweg have only one page on Digby.

⁸ Cf. Digby, *op. cit.*, first treatise, p. 426.

sets out to explain a philosophy that "takes the same solid way which Aristotle walked in before".⁹ But Digby is not a dogmatic Aristotelian. He is too deeply impressed with the "new science" and Descartes' "new way of ideas". Personally acquainted with Mersenne, Fermat, Descartes, and other philosopher-scientists of his time, and an ardent admirer of Gassendi, Gallileo, and Bacon, he refers to all these men with utmost respect.¹⁰ So he does not reject their views out of hand. Rather, he tries to combine their new ideas with his basic Aristotelian commitment. This mixture of Aristotelianism and modern philosophy is characteristic of Digby and determines his main problems: 1. the reconciliation of Aristotelian physics with modern atomism; 2. the defense of the immortality of the soul against atomists, Aristotelians, and skeptics; 3. the refutation of skepticism through the discovery of indubitable truths; 4. the outline of a theory of knowledge that incorporates Descartes' new way of ideas.¹¹ The last problem leads to his doctrine of notions.

All knowledge is notional. This, in one sentence, is Digby's rebuttal of the ideists whom he takes to hold that knowledge is by way of ideas about material substances.¹² His argument, as one

⁹ *Ibid.*, p. 424.

¹⁰ Digby met Mersenne around 1636 and became a member of the circle around Mersenne. Cf. Petersson, *op. cit.*, and M. Nicolson, "The early stages of Cartesianism in England", *Studies in Philology*, 1929, 26 pp. 356-74. He met Fermat in 1646. Cf. Remusat, *op. cit.*, vol. 1, p. 300. According to the same author, he met Descartes in Holland.

¹¹ Digby holds that Aristotle was an atomist. Cf. his first treatise, p. 425. That Digby was thought of as an outstanding proponent of atomism can be seen from the fact that his name appears together with those of Gassendi, Democrit, and Boyle in the title of a book by Henricus Maius, *Physicae veteris noviter adornatae ad principia Democriti, a Gassendo, Verulamio, Boylio, Derodone, Digbaeo aliisque redintegrata synopsis*, Frankfurt 1689.

For some comments on the problem of the immortality of the soul in the 16th and 17th centuries see Robert Lenoble, *Mersenne; ou la naissance du mécanisme*, Paris 1943. For an excellent account of skepticism in the 17th century see R. H. Popkin, "The sceptical crisis and the rise of modern philosophy", *The Review of Metaphysics*, 1953-54, 7, pp. 132-51, pp. 308-22, pp. 499-510.

¹² Cf. Digby, *op. cit.*, first treatise, p. 15.

might expect, depends, first, upon his conception of knowledge and, second, upon his meaning of 'notional'. A few words will suffice to explain the former; the latter will occupy us for the remainder of this section.

According to Digby, only science constitutes knowledge. However, if science were about material substances, as the Cartesians presumably contend, then there could be no knowledge at all; for if it also be true that only ideas are ever given to the mind how could one compare ideas with the substances of which they are likenesses?¹³ This question, we know, occupied the followers as well as the critics of Descartes. Berkeley's immaterialism is one kind of solution. Long before Berkeley, Digby realized that and proposed a solution similar to his. The heart of this solution is his doctrine of notions.

For Digby, all knowledge is contained in categorical propositions.¹⁴ Take as paradigm the form 'all *F* are *G*' or, as we would write, combining Hume with PM logic, ' $(x) (F(x) \supset G(x))$ '. Digby, of course, did not propose a Humean analysis of causation. Nor did he grasp the significance of predication; for he held that all categorical propositions are statements of identity.¹⁵ Laws, in his view, are categorical propositions expressing identities.

The descriptive terms of categorical propositions are predicates. Digby calls their referents notions.¹⁶ This is one-half of the explication of his thesis that all knowledge is notional. The other half is his insistence that categorical propositions express knowledge if and only if they are about physical objects. *Thus all terms of scientific discourse refer to notions, not to phenomenal objects (ideas).*¹⁷ But Digby also accepts the Cartesian turn: only ideas are immediately given to the mind. What then is the connection between ideas and notions? We noted that he rejects the Cartesian answer. Ideas, for him, are not likenesses of material

¹³ *Ibid.*, second treatise, pp. 3-4.

¹⁴ *Ibid.*, pp. 28-29.

¹⁵ *Ibid.*, p. 14 and p. 21.

¹⁶ *Ibid.*, p. 14.

¹⁷ *Ibid.*, p. 54 and pp. 59-60. His view is expressed by the statement *Nihil est in intellectu quod fuit prius in sensu.* (p. 54).

objects. So he offers an alternative similar to Berkeley's. Before turning to it, we must take care of one point.

Digby is a nominalist. All existents are said to be particular and definite. To avoid some well-known difficulties, he embraces the doctrine of common names.¹⁸ Strictly speaking, his nominalism has two levels. In regard to *sensa* he holds, at least implicitly, that characters do not exist; their names are common names of particular sense data.¹⁹ In regard to notions he states explicitly that their names are common names of particular physical objects. I shall not and need not here decide whether the common name doctrine solves the problems of nominalism. For expository purposes, though, I shall assume that it does. This understood, we are prepared to explore the connection between ideas and notions.

Notions are "products" of the mind's "transformation" of sense experience.²⁰ In this sense, they are constructs out of *sensa*. Specifically, all notions are formed by mental acts of comparison, where the comparison is among *sensa*.²¹ All notions are thus "relational" or "respective".²² Complex notions are built from simpler ones through identity statements. I submit that in these and similar assertions Digby dimly anticipates the modern idea of definitional reconstruction. Let me explain.

Ever since Berkeley, philosophers have been familiar with attempts to analyze statements about physical objects in terms of *sensa*. 'Chair', for instance, becomes upon such analysis a defined predicate. Schematically, the definition replaces 'chair' by 'A and B', where 'A' refers to what has been called the sensory core of a chair percept while 'B' abbreviates the conjunction of a number of laws.²³ 'A' is in turn a conjunction of statements. Some

¹⁸ *Ibid.*, pp. 10-12.

¹⁹ I use '*sensa*' to refer to particular sense data (individuals) as well as their characters (universals).

²⁰ *Ibid.*, p. 59.

²¹ *Ibid.*, p. 7 and p. 9.

²² *Ibid.*, p. 9 and pp. 53-54. There are, however, two innate notions which are not relational. See below.

²³ For a recent defense of definitional reconstruction see G. Bergmann,

of these latter assert that certain descriptive relations, e.g., spatial ones, obtain among the sense data of the core. Crude as it is, the schema thus shows that a definitional reconstruction contains statements about *sensa*, statements about descriptive relations among sense data, and statements of law. A law states what is also called a "causal relation". Upon a Humean analysis this particular "relation" turns out to be logical.²⁴ Be that as it may, a "causal relation" is something very different from what we now mean by a descriptive relation. It will help my purpose, though, if I sometimes use 'relation' so broadly that a relation may be either descriptive or causal.

Digby does not, like the moderns, reconstruct the predicate 'chair'. Rather, he calls the chair itself a notion, i.e., a product of the mind's working, by acts of comparison, on *sensa*. Knowledge, it follows, is not about material substances, but, rather, about mind-produced patterns of *sensa*.²⁵ Patterns, of course, are relational. The descriptive relations in the pattern "chair" are, according to Digby, established by acts of comparison. That is why I said that he anticipates the modern idea of reconstruction. About the "causal relations" which are mentioned in the reconstruction of 'chair', he is not so clear. That is why I called his anticipation dim.

But now a new question arises. If Digby thought of notions as relational patterns of ideas, why did he not simply introduce the dichotomy of simple and complex ideas? The answer has two parts. He holds, first, that *descriptive relations do not exist*.²⁶ Unlike ideas, they are not given to the mind, but are products of mental acts of comparison. To grasp this point, consider a simple relational situation, say, a green spot being to the left of

"The revolt against logical atomism (Part 2)", *The Philosophical Quarterly*, 1958, 8, pp. 1-13.

²⁴ More specifically, it is a defined logical relation. Its definition reads " 'R_c(F, G)' for '(x) (F(x) ⊃ G(x))' ".

²⁵ Such "patterns" are, according to Digby, *in the mind*. Since a notion is literally identical with a physical thing, the thing is truly *in the mind* when we know it. Cf. Digby, second treatise, p. 3, p. 51, p. 53.

²⁶ *Ibid.*, pp. 6-7.

a red spot. A realistic account lists three elements, the two spots and the relation to-the-left-of. Not so Digby's analysis. It too yields three constituents, but the third is a mental act of comparison, not a relation given to the mind. Different acts of comparison generate different relations among the spots. His first reason, then, for calling his patterns notions is that they contain descriptive relations which are not *given to* but are *products of* the mind.

Whenever the mind, working on *sensa*, forms relational notions, it establishes a firm ground for knowledge by further contributing *the two "innate notions" of substance and existence*.²⁷ In other words, existence and substance are, according to Digby, two further ingredients of the pattern "chair"; and they too are contributions of rather than, like ideas, given to the mind. This is the second reason why Digby cannot consistently think of physical objects as complex ideas. It is worth noticing, though, at least in passing, that a thinker as acute as Digby is content to introduce substance as an "innate notion", presumably unanalyzable, and let it go at that.²⁸ This curious gap, I submit, corresponds to his failure to grasp clearly the role of causal relations in the reconstruction.

To sum up. Notions, though constructed from *sensa*, contain ingredients not given in sense experience but contributed by the mind, namely, descriptive relations and the two notions of existence and substance. This is Digby's notion of notions. It suggests three questions.

1. I spoke repeatedly of ideas as given, of notions as con-

²⁷ *Ibid.*, p. 5, p. 9, pp. 51-52, p. 59.

²⁸ Digby asserts that all of Aristotle's categories are really notions. Cf. first treatise, p. 41; second treatise, p. 9. A very similar view is held by Richard Burthogge. Cf. *The philosophical writings of Richard Burthogge*, ed. by M. W. Landes, Chicago and London 1921. Geulincx interprets Aristotle's categories similarly. Cf. *Metaphysica ad mentem Peripateticam*, introd., sect. 2. In *Opera Philosophica*, rec. J. P. N. Land, Hagae Comt., 1892, vol. 2, p. 204. It would therefore be of interest to determine whether Geulincx knew Digby's book. Since it seems probable that Burthogge became acquainted with Geulincx's philosophy while he studied at Leyden, a line of influence could perhaps be traced from Digby through Geulincx to Burthogge.

structed. Some philosophers use 'exist' so that what is "given" exists, while what is "constructed" does not. Does Digby fall into this pattern? The answer is Yes. He holds that notions do not exist, his reason apparently being that what is constructed from existents (ideas) together with nonexistents cannot itself be an existent.²⁹ The difference between ideas and notions is thus ontological. *Ideas are existents, notions are not.* Hence the need for the new term 'notion'.

2. Ideas are existents. Descriptive relations are decomposed into ideas and acts of comparison. Do such acts exist? Digby, as far as I could discover, gives no explicit answer. But one could reasonably argue that one who proposes his analysis of relations holds, at least implicitly, that there are at least two kinds of existents, namely, ideas and acts.

3. If there are acts of comparison, are there also acts of sensing, that is, acts which are awarenesses of ideas? Again, Digby provides no answer. Taking a hint from Sergeant, however, one may assume that a doctrine of notions in the style of Digby can consistently distinguish between acts of sensing and things sensed. If so, then there are at least, two kinds of acts, namely, acts of comparison and acts of sensing.

II

What I wish to say about Berkeley depends on two assumptions. Since I cannot defend either in this paper, I shall at least indicate that I am aware of some objections that have been raised against them. First, I assume that Berkeley uses 'idea' to refer to phenomenal and not to physical things. This assumption, if assumption it be, agrees with most interpretations of Berkeley. Yet, Bracken has recently urged us to distinguish two trends in Berkeley.³⁰ According to one trend, ideas are *sensa*. According to the other, they are physical objects. My second assumption is perhaps not as widely shared as the first. But in the light of Luce's and Jessop's scholarly discussions it seems to me undeni-

²⁹ Cf. Digby, second treatise, preface. Also, p. 9.

³⁰ Cf. H. M. Bracken, "Berkeley's realism", *The Philosophical Quarterly*, 1958, 8, pp. 41-53.

able that Berkeley's earlier and later writings form a system without essential break.³¹ If there is a difference between the earlier and the later Berkeley, it is one of mood rather than of viewpoint. I shall therefore draw my documentation from the *Siris* as well as the *Principles* and the *Dialogues*.

Berkeley, unlike Digby, distinguishes two kinds of knowledge. One he calls ideal knowledge or natural philosophy; the other, notional knowledge.³²

Ideal knowledge or science does not consist in mere acquaintance with *sensa*. According to Berkeley, the senses as such know nothing.³³ Rather, science begins with the discovery of general rules or laws.³⁴ To know means to know laws and hence to be able to predict which *sensa* will follow which others.³⁵ Without laws there is thus no ideal knowledge. In this respect there is full agreement between Digby and Berkeley. One minor difference concerns the form of laws. Digby thinks of them in Aristotelian fashion as categorical propositions expressing identities. And identity, I now add, is for him nothing but the notion of existence. Berkeley, on the other hand, seems to hold that laws are rules of the form "whenever so-and-so, then such-and-such".³⁶ More striking is a difference that touches the core of our problem. Digby asserts that all scientific knowledge is notional. Berkeley calls science ideal knowledge. Does the difference in terminology indicate a substantive difference? The answer is Yes and No. Or, rather, it has two parts.

First. Digby called science notional because it involved descriptive relations and such relations are known notionally, not by way of ideas. Moreover, science, for him, is stated in laws and laws, we just remarked, are identity statements, identity being

³¹ Cf. A. A. Luce, "The unity of the Berkeleian philosophy", *Mind* 1937, 46, pp. 44-52, pp. 180-90. Compare also Jessop's introduction to the *Siris* in *The works of George Berkeley, Bishop of Cloyne*, 1953, ed. by A. A. Luce and T. E. Jessop. All references will be to this edition.

³² Cf. *Principles*, sect. 101; *Siris*, sects. 247, 264, 285.

³³ Cf. *Siris*, sects. 253, 305.

³⁴ Cf. *Principles*, sects. 30, 31, 62; *Siris*, sects. 231, 234, 247, 252.

³⁵ Cf. *Siris*, sects. 252, 253.

³⁶ Cf. Berkeley's view on the connection between sign and thing signified.

expressed by the "innate" notion of existence. According to Berkeley, all general rules (causal relations) are known notionally. The laws of science express universal intellectual notions.³⁷ Also, all descriptive relations are notions.³⁸ Furthermore, Berkeley seems to realize that science requires descriptive as well as causal relations. It follows that by Digby's criterion for notional knowledge the kind of knowledge Berkeley calls ideal is really notional. More sweepingly, one may conclude that in Berkeley all knowledge is notional and that there is, strictly speaking, no ideal knowledge at all.³⁹ In this sense, Berkeley speaking of ideal knowledge and Digby speaking of notional knowledge merely use different phrases for the same thing.

Second. If one probes more deeply, one comes upon a very real difference. Why do Digby and Berkeley both hold that some things are known notionally? I stated what I take to be Digby's reason: whatever does not exist can only be known notionally; and conversely. Berkeley's reason is different. To see that it could not possibly be Digby's requires but a moment's reflection. Berkeley holds that spirits and acts are known notionally.⁴⁰ But spirits and acts, in his philosophy, are existents.⁴¹ Hence, to be known notionally and to be a nonexistent cannot be one and the same thing. Specifically, in regard to relations, Berkeley seems to admit that causal relations, which are known notionally, are existents. For, he holds that laws of nature are "given" by God and discovered by the understanding.⁴² Not contributed by the human mind, they constitute, as it were, God's language. Descriptive relations, on the other hand, do not seem to be existents in his system.⁴³ I say seem because Berkeley himself has never ex-

Principles, sects. 30, 65.

³⁷ Cf. *Dialogues* (*Works*, vol. 2), p. 173; *Siris*, sect. 264.

³⁸ Cf. *Principles*, sects. 89, 142.

³⁹ Bracken arrives at the same conclusion. His argument, however, is different. Cf. Bracken, *op. cit.*, p. 52.

⁴⁰ Cf. *Principles*, sects. 27, 89, 142.

⁴¹ Cf. *Principles*, sects. 27, 138, 139; *Dialogues* (*Works*, vol. 2), pp. 231-32.

⁴² Cf. *Principles*, sects. 30, 32.

⁴³ Cf. *Principles*, sect. 142; and Jessop's footnote on page 106.

pressed himself unambiguously on this point. But I am prepared to argue that, in the spirit of his system, descriptive relations do not exist.⁴⁴ In this paper I shall therefore assume that they don't. The last point I shall make depends on this assumption.

Negatively, we have convinced ourselves that Berkeley's notional knowledge cannot be rooted in the existent-nonexistent distinction. Positively, it remains to discover its source. It will greatly simplify our task if, disregarding other selves, we consider the knowledge each has of his own self and its acts.

According to Berkeley, relations as well as selves and acts are only known through acts.⁴⁵ This is the cue to the nature of notional knowledge. *Whatever is known notionally, existent or not, is known through acts. Conversely, whatever is not known through acts is not a notion.* To be a notion or to be known notionally means no more nor less than to be something known or knowable through acts. This, I submit, is the root for Berkeley's distinction between ideas and notions. Digby's we remember turns on the dichotomy existent-nonexistent. This is the crucial difference between our two authors. Let me next consider two likely objections to what has just been said about Berkeley.

If notional knowledge is knowledge by way of acts, there can be no acts of sensing. Otherwise acquaintance with *sensa* would have to be what according to Berkeley surely it is not, namely, notional. But does not Berkeley distinguish between act and content in regard to *sensa*? This is the first objection. My answer is that Berkeley does not make this distinction. Rather, he rejects acts of sensing.⁴⁶ If this is so, then the fundamental difference between ideas and notions can consistently be what I claim it is, namely, that the latter are and the former are not known through acts.

⁴⁴ Berkeley states that the mind imparts order and harmony "to the phenomena which we term the course of nature". Cf. *Siris*, sects. 293, 295, 347. It is, however, possible that he here refers to God's mind.

⁴⁵ Cf. *Principles*, sect. 142; *Dialogues* (*Works*, vol. 2), p. 233.

⁴⁶ Cf. *Dialogues* (*Works*, vol. 2), pp. 194-97. This is convincingly argued by Doney. Cf. W. Doney, "Two questions about Berkeley", *The Philosophical Review*, 1952, 61, pp. 382-91.

If one maintains, as I do, that Berkeley denies the existence of acts of sensing, must not one conclude that contrary to Berkeley the mind is passive in perception? This is the second objection. The answer is again negative. One can without contradiction hold two things, namely, first, that there are no acts of sensing and, second, that the mind is active in perception. For, perception involves not only sensing but also reasoning and inference; and in reasoning and inference the mind is active.⁴⁷ Since this view is thus consistent, it makes at least sense to attribute it to Berkeley. I am persuaded that it is the one he actually held.

Recall the thesis. In Berkeley's system, to be a notion and to be known through an act is one and the same thing. This is the thesis I just defended against two objections. Now I wish to support it by a further consideration. Assume, for the sake of the argument, that there are in the system acts of sensing. Then ideas as well as notions are known through acts. If so, why did Berkeley insist that relations are notions rather than ideas? There are three possible answers. 1. Ideas exist, relations do not. 2. Ideas are passive. 3. There is an intrinsic difference between acts of sensing and those others, Digby calls them acts of comparison, through which we know relations. None of these answers will do. 1 won't do because spirits are existents as well as notions. 2 won't do if, as I have assumed, descriptive relations are not existents; for only what exists can be either active or passive. 3 won't do because Berkeley nowhere makes the required distinction between the two kinds of acts. What will do, though, is the distinction I proposed: Ideas, unlike relations and all other kinds of notions, are not known by acts. I take it, then, that Berkeley distinguishes between knowledge through acts and, as I shall call it, through acquaintance. The distinction coincides with that between notional and nonnotional (though not, as he himself uses the phrase, ideal) knowledge. The motive for the latter is the commitment to the former. Without the former, Berkeley would not have needed the latter.

⁴⁷ Luce does not seem to appreciate this point. Cf. A. A. Luce, "Berkeleyan action and passion", *Revue Internationale de Philosophie*, 1953, 7, pp. 3-18.

What remains to be said is again best introduced by a question. Why did Berkeley think it necessary to distinguish between knowledge through acts and through acquaintance? In other words, why did he not collapse the two by either (a) admitting ideas of selves and acts, or, alternatively, (b) by admitting acts of sensing?

(a) Berkeley rejects ideas of selves and acts because he believes that there can be no ideas of active things.⁴⁸ No matter what inner sense we invoke, all it provides are inactive ideas.⁴⁹ As Popkin suggested, this emphatic insistence on the activity of spirits may well be a reaction against Malebranche.⁵⁰ If this is right, then Berkeley's metaphysics of notions exhibits the following pattern. 1. In order to avoid Malebranche's conclusion, Berkeley regards human minds as active efficient causes. 2. Ideas, on the other hand, are held to be inactive; they are not causes at all. 3. Ideas as well as minds exist. 4. Their having the same ontological status corresponds to their both being immediately known.⁵¹ 4. Their difference in regard to activity-passivity corresponds to the ways in which they are known or given. Ideas are known by acquaintance; selves and acts, through acts.

(b) Berkeley could have introduced acts of sensing, provided only that he also introduced an intrinsic distinction between these acts, on the one hand, and so-called reflex acts, on the other. In fact, I can think of no textual evidence to exclude this possibility. But I can think of a reason for his not having done so that is at least in the spirit of the system. Given two kinds of existents, selves and their ideas, acts of sensing are deemed somehow to "connect" the two. What, one may ask, is the nature of this connection? One classical answer has it that acts of sensing are relational. Being relational, they will not be logical, but

⁴⁸ Cf. *Principles*, sects. 142; *Dialogues* (*Works*, vol. 2), p. 231.

⁴⁹ This excludes Husserl's interpretation of Berkeley's view on notions. Cf. E. Husserl, *Logische Untersuchungen*, Halle 1922, vol. 2, p. 176.

⁵⁰ Cf. R. H. Popkin, "The new realism of Bishop Berkeley", in *George Berkeley*, Univ. of Cal. Publications in Philosophy, Berkeley and Los Angeles 1957.

⁵¹ Cf. *Dialogues* (*Works*, vol. 2), pp. 231-32.

surely must be descriptive. If Berkeley himself entertained this possibility and if, as I have assumed, he denied that descriptive relations exist, then he could not but reject sensory acts.

There remains the fact that the system as it stands indubitably contains acts, though not of course, if I am right, acts of sensing. Again, if I am right, Berkeley must consistently hold that all acts are nonrelational. Thus he must eventually face the question how a nonrelational act can "connect" with its content. The question is difficult indeed.⁵² In the nature of things, one cannot be sure that Berkeley actually asked himself this question. I believe, though, that had it been put to him, he would not have known what to say in reply. The reason why I believe this leads us back to the problem of notions or, what amounts to the same, to the problem of notional knowledge. Once more, if I am right, notions are those and only those things that are known by acts. Berkeley, every one agrees, says very little, much less than one would wish, on this crucial point. Is it then not reasonable to assume that what kept him silent was that he felt a difficulty and that, structurally at least, this difficulty was the difficulty of the non-relational act?

⁵² For a recent attempt to answer it, see G. Bergman, "Intentionality", in *Semantica* (Archivio di Filosofia), pp. 176-216.

Uses of Language and Uses of Words

With Application to a Problem of Frege¹

by

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I

Contemporary philosophers have come more and more to put questions about language in terms of "use". The effect has, on the whole, been salutary, for reasons I do not here aim to discuss. But it is worth pointing out that there is a rather far-reaching ambiguity on "use". This ambiguity, while easily resolved, poses an important question over the relationship between the two uses of "use".

The ambiguity is indicated, on the one side, by the phrase "the fact-stating (or statement-making) *use of language*", and, on the other side, by the phrase "the referring *use of words*". I thus distinguish these two ideas by calling the first "use of *language*" and the second "use of *words*", where any kind of expression, and not words alone, may be given the same word use.

A use of language is a category of what we can call language acts. It is a classification according to what the agent aims to produce and will produce if his act succeeds. Stating is a kind of use of language. It comprises all those acts in which the agent aims to produce a statement. The statement, which might have been made *via* and identified in any number of language acts -- call them "assertions" if you wish --, is what a competent observer could identify in the agent's act, if that act is successful.

¹ This paper was read before the 32nd annual meeting of the Pacific Division of the American Philosophical Association, at Eugene, Oregon, December 30, 1958.

The act will in fact succeed provided the agent makes such movements as will enable others to identify what he aims to say, but that is a very tricky thesis and I do not propose to discuss it here.

A use of words is a category of features by reference to which what the agent says might be partially identified. Referring is a use of words. In making a statement, say, one may find it necessary to refer to this object or that as identifiable in a certain way, and to identify what was said we must ascertain that that particular reference was made. If the agent succeeds in his language act, I shall say that he realizes an example of the use of words in question, e.g. he *realizes* a referring use of words. Now, if the act does succeed, the agent will produce (e.g.) a statement. In doing his act he will, by realizing (e.g.) a referring use of words, make the statement identifiable in some special respect; and I shall say that in that case he contributes (e.g.) a reference to the statement. Thus, should I state that $\sqrt{2}$ is irrational, I would refer to $\sqrt{2}$, thereby contributing to the statement a reference to $\sqrt{2}$.

Here I shall pretty much confine myself to language acts, whether or not successful, considering them as categorized under uses of language, which may be sub-divided, e.g. according to form (*subject-predicate stating* would be such a sub-division). I shall, that is, consider not (e.g.) statements or promises, but acts of stating or promising.

Uses of language are enormously numerous. Representative examples would be statement-making, promise-making, ordering, contracting, counting, baptizing, marrying, swearing, pronouncing criminal sentence, requesting and questioning.

Uses of words, also unlimited, are more multifarious in character than are uses of language. Some are restricted to this or that use of language, others to certain subject-matters, while others still are "topic neutral", and uses may be "topic neutral" in fulfillment of many different tasks. Words are used to refer, to distinguish, to relate, to qualify; they may be used as conjunctions, connecting statements by connecting sentences, or marking off the conclusions of arguments; they may be used to indicate that a certain kind of statement is to be made, where

statements may be classified into many different kinds, in many different ways; or that a certain use of language is being attempted; they may be used to hold open gaps, signal cross-references; to indicate the mode of utterances and where they start and stop, and how definitively are to be taken. There is much crossing and criss-crossing here.

In what follows I shall for purposes of convenience mostly restrict myself, on the one side, to the referring use of words, and I hurry to grant that what I say about referring will not hold generally of all uses of words, though I should hope that something comparable could always be said.

On the other side, I restrict myself mostly to the statement making use of language, and, in particular, to what I shall call "predicating". We predicate when we aim to make a singular subject-predicate statement. In English, we frequently do that by uttering or inscribing upon an appropriate occasion a sentence of the form "... is (a) ***", where "..." is to be replaced by a referring expression and "***" is to be replaced by a suitable common-noun or adjective, i.e., by a word used to characterize and distinguish objects logically like the one referred to. Thus when talking about material objects we may put in place of "***" size, shape and colour adjectives, among others.

To predicate successfully *is* to make a subject-predicate statement; predicating is a sub-use of language, a species of language act. In order to predicate successfully we must somehow or other contrive to refer to an object and say something about it, without having to refer to any other object; otherwise we fail at the task.

Referring is not a use of language, but a use of words, one of the subsidiary elements contributing to the success of language acts falling under different uses of language; while predicating is fact-stating, referring is neither fact-stating, command giving, nor anything else comparable.

This may be seen by observing that a referring expression may be used in one and the same way now to make a statement, now to ask a question and now to forward the success of acts falling under any one of innumerable uses of language. So, while we are

not invariably bound to refer when using language (we seldom do so when making universal statements), referring still may be called for when we employ language in innumerable different uses.

Presently I shall attempt to show we refer *only* when attempting some language act falling under a use of language. Granting this, what we see is that, if referring were a use of language, it would also always involve the discharging of some *other* use of language. But these other uses of language are not related among themselves in any way at all like that. Asking a question is not making a statement, nor anything else. This begins to indicate that referring is not itself a proper use of language.

Notice now, that we do not make a statement, nor accomplish anything else comparable to making a statement, merely by calling off a list of names. When we merely refer, and nothing else, we are not yet doing anything. Only that is a misleading way of putting it: in fact there is no such thing as "mere referring".² We refer only if we do something of a conventional kind with the object ostensibly referred to, e.g., predicate of it such and such property. This is crucial: we fail even to refer with a referring expression unless we put it into the context of a sentence or by some other means say something about the referent as identified -- order it, welcome it, etc. etc. Take, for example, a referring expression, a surname like "Wittgenstein". If I pronounce the word "Wittgenstein" that might be construed as a summons or as an answer to the question "Who is the greatest modern philosopher?" But if I simply say "Wittgenstein", not intending to do anything with the word except to refer to Wittgenstein, then I fail to do even that; I am not referring to Wittgenstein, although the word "Wittgenstein" is tailored for the job. If, however, I say, "Wittgenstein is Viennese", then I do indeed refer to Wittgenstein, and at the same time I say something about him. An important fact, then, about referring is that a word is not yet used to refer unless it is used

² I mean as a use of *words*; there is a use of *language* we might call "giving references" towards the success of which we must use words to refer, and in other ways as well.

to forward the success of some language act classifiable under a use of language. Regard it so: generally, to have mastered a use of words is to know how to employ conventionally chosen devices in furtherance of various uses of language. When a meaningful word – – and all words are meaningful – – is successfully used, it actually contributes to the success of an act falling under one of these uses of language. Hence there can be no such thing as “mere referring” or “mere full-stopping” or the like. We refer always in the act of doing something with language; we can full-stop only when we have a language act to stop.

Referring to something might be regarded as an “element” of a successful language act. If the success of the act requires that element – – as the making of a subject-predicate statement requires that one refer to a subject – –, and that element is absent, then the act will fail. The same element may be common to many different acts. If all the elements are the same in two different acts, they will produce the same upshot. We might make the same statement or ask the same question many times. If only some of the elements are common to two acts, we may expect their upshots to be different but related in specific ways. Thus, if reference is made to the same thing in two statement-making acts, the statements emerge, if the acts are successful, will have the same subjects. Now, while the same element might contribute to many language acts, it must always be the element of some language act, and cannot be found apart from any and all such acts.

We may, of course, refer without using a referring expression. The language element might be present in the act, although no word marks its presence. Thus we might state that John has gone home by replaying “home” to the question, “Where has John gone?” The language situation, itself never dispensable, will often stand in for the use of certain words which, on other occasions, we should use in making such and such statement, asking such and such question, etc.

The thesis I am arguing for was long ago anticipated by Wittgenstein’s application of Frege’s dictum that “*nur im Zusammenhang des Satzes hat ein Name Bedeutung*”, and has recently

been elaborated by Ryle, who observed that it is not correct to speak of using sentences, as it is correct and useful to speak of using words.³

In the *Tractatus*, Wittgenstein appropriated Frege's dictum partly to indicate how far Frege went wrong in supposing that, while words occur meaningfully in sentences, sentences themselves are just another kind of name. Unfortunately it is not perfectly clear whether Wittgenstein's "Satz" is best rendered as "sentence" or "statement". If he meant "statement", then, of course, he went slightly wrong, for words are used not only to make statements but also to ask questions, etc. But if Wittgenstein meant "sentence", then he went slightly wrong again, but for a quite different reason. Single words will sometimes suffice to make statements. More generally: We may do without sentence contexts if other elements in the language situation secure that we shall succeed in accomplishing what we would accomplish with a full-fledged sentence.

Ryle's formulation of the distinction is, also, somewhat inaccurate and probably inoperative. The fact is that we have yet to hit upon a generally workable formula for separating sentences from words, not to mention morphemes in their many kinds. As we commonly employ "word" and "sentence", the distinction between them is far from clear and certainly not sharp; nor is it obvious that we can make the separation as precise as we would wish and still retain it as a useful philosophical tool (Are we to say that "yes" is a sentence or a word; or — as is more likely — is it both? Is "gl" a word? No, but it has a sense, and so presumably a use, as in "glisten".) I do not deny there is this difference; I am sure there is. But I am confident that the distinction between word and sentence, for what it is worth, cannot be drawn except in terms of what one does with language.

³ Frege, *Grundgesetze der Arithmetik*, v. 1, nos. 29–31, *Foundations of Arithmetic* (Tr. by Austin), pp. x, 73 ff., 115; Wittgenstein, *Tractatus*, 3. 3; Ryle, "Ordinary Language", *Phil. Rev.*, April, 1953, pp. 178 ff. See also Wittgenstein, *Philosophical Investigations*, Part II, ii. Strawson has queried the point in his review of *Philosophical Investigations*, *Mind*, 1954, p. 73. Since this was first set down it has become quite a journal topic.

One might, for example, as a first approximation, want to say that a sentence is an expression of some tongue cut out for completing language acts; a word might then be explained as an unbound expression (not, e.g. an affix) of some tongue which may, by the rules of that tongue, be employed in connection with other expressions to complete language acts. This is inaccurate, and certainly not designed as a definition, but it is enough to indicate that we must make a forward reference to the use of language in order to draw the distinction between a word and sentence. But let us anyway suppose that we have this distinction between a word and a sentence: what good will it do us? It seems pretty clear, as against Ryle, that one might occasionally wish to inquire whether a certain sentence were used (e.g.) to make a statement or to give an order. It is, of course, true that in such a circumstance we would be doubtful, not about a use of words, but rather, about a use of language. All these observations indicate that Ryle comes onto the desiderated distinction from the wrong position, from expressions rather than from the uses they have; indeed, he draws the distinction at the wrong level, at the level of repeating expressions, when it must be drawn at the level of using language, e.g. making statements. It is this, I believe, which accounts for the resistance his doctrine has encountered in almost all quarters. Still, most of what Ryle says here seems to me to be substantially right.

To form a sentence thereby succeeding in a language act *is* to use words, and there is no further room to speak of using the sentence as a kind of complex word. Only that is a misleading brachylogy. More accurately: putting sentences together on appropriate occasions so as to make statements, etc. *is* to use words. Ryle correctly observed that we learn the rules governing word-usage, and we do not then need to learn new rules governing the usages of sentences. Learning how to construct meaningful sentences is involved in learning the rules for employing words.

II

I have been telling a modern version of an old story. It dates at least from Plato, and was a favorite of Cartesians and Idealists. It has many lessons, philosophical applications if you wish, especially for the Modern Man. I shall go into but one of these, then detailing only a highly local case. I wish to apply what has been said to a kind of platonizing.

My argument shows that the distinction between stating and referring is not a metaphysical distinction at all, neither, *a fortiori*, is the distinction between predicating and referring a metaphysical distinction.

While we can represent a variety of predication by signs like "is white" or "() white" or "white x", these do not have uses in the way in which "white" or "Polly" do. They schematize rather the application of (e.g.) "white" to objects, and thus a family of completed language acts. It follows from this that the distinction between predicating and referring is not a distinction between kinds of things. Metaphysical distinctions are to be drawn kinds of objects to which we might refer, objects such as material objects, colours, numbers and – – I add – – statements, *after* they have once been made.

The many philosophers (Plato, Moore, Russell, Frege, Husserl, and Meinong among others) who have supposed that, when we make a statement about an object, we refer not to that object alone, but also necessarily to another object, like a proposition, truth-value, *Sachverhalt* or *Objektive*, have simply failed to grasp the distinction between a use of words and a use of language. To give an extended illustration of the point, I shall now try to show that Frege's celebrated difficulty over Concepts flows from the same source.⁴

⁴ For Frege's theory see *Translations from the Philosophical Writings of Gottlob Frege*, ed. by Geach and Black, esp. "Function and Concept", but also "Concept and Object", "What is a Function", and in other articles pp. x, 48 ff, 63 ff, 108; *Grundgesetze*, v. I, nos. 1-4, 8, 19, 21-26, and "Ueber die Grundlagen der Geometrie", *Jahresber. d. d. Mathematiker-Vereinigung*, v. 12, pp. 371 ff.

Frege observed that it is never enough merely to refer to an object in order to say something about it; we must also say something about it. Expressions used to refer to Objects, Frege called "*Eigennamen*", which corresponds in a rough way to the more recent "referring expressions", which I shall use here.⁵ Expressions used to talk about objects are properly included in the class of what Frege called "*Funktionnnamen*", where this class seemed to comprise all meaningful signs not *Eigennamen*, exclusive (it would appear) of the assertion sign. One-place Fregean *Funktionnnamen* used to make statements I shall call "predicates" and, following Frege, shall think of them as written with a gap. Thus "() blue" is a predicate. I shall restrict myself to predicates as they relate to the making of statements.

Now there is a certain propriety in insisting that a fully meaningful referring expression, meaningfully employed, must refer to an object, and Frege did so insist.⁶ Frege docketed the Object referred to as the *Bedeutung* of the referring expression, which latter we may now assume to be filling the gap of a predicate. What now is to be the *Bedeutung* of the predicate, for surely it must have one? ⁷ Frege answered that it was a Concept (*Be-*

⁵ Rough, because a referring expression need not refer as must a fully meaningful *Eigenname*. Frege did allow that an *Eigenname* might be deficient in meaning, having only Sense (*Sinn*), e.g., "Pegasus". But this is another matter. Frege, moreover, included among *Eigennamen* things not properly classified as referring expressions, e.g. sentences. It may be well to warn that the present discussion touches only incidentally on Frege's distinction between *Sinn* and *Bedeutung*, being solely concerned with the *Bedeutungen* of expressions, and the view that every fully meaningful expression must have one.

⁶ A referring expression may have a use and still not be used successfully on some given occasion to make a statement, perhaps because it has no referent. Then the whole act fails.

⁷ I shall not argue the disputed question whether Frege would have held that Functions or, in particular, *Begriffe* are *Sinne* or *Bedeutungen* of *Functionnnamen*. The distinction between Function and Object antedates that between *Sinn* and *Bedeutung*, and indeed it seems that Frege simply never considered the question on which side of the latter distinction his Functions would fall. It is interesting how Frege almost prohibits us raising for Functions the issues over identity which more than any other brought him to

griff), but was not long in deciding that Concepts must be very different from Objects, these last being the proper *Bedeutungen* of referring expressions. I suppose his reasons are those we have observed before: we get no statement simply by calling off a list of names. In order to get a statement about an object we must refer to that object and predicate something of it. We have seen that predicating is indeed very different from referring; and one of the main reasons is that we must always predicate *of*. Frege accordingly thought that the *Bedeutung* of a predicate must directly embody this essentially predicate character, and held that this *Bedeutung*, like the predicate itself, must have a gap; as he put it, the Concept is "*ergänzungsbedürftig*" or "*ungesättigt*".

Frege's puzzle was then how it is that we can ever manage to talk meaningfully about Concepts. We can talk about something only by referring to it; that is, by using a referring expression; but what we refer to must always be an Object, and never a Concept, for Concepts are the *Bedeutungen* of predicates. The attempt to talk about Concepts is inevitably self-defeating, for we can talk only about things to which we might refer; and Concepts are excluded, almost by definition.⁸

Now, rightly interpreted, Frege was fully justified in insisting upon Concepts; rightly interpreted, he is telling us that we cannot get by without predicates; referring expressions are not enough. Making allowances for the fact that predicating is not the only species of stating, it is manifestly clear why we need

the contrast between *Sinn* and *Bedeutung*, for he expressly stated that Function-names cannot meaningfully occur on either side of "=". It is also interesting if expected that all his examples of *Sinn* are Objects. I shall simply take it that for Frege Functions and, in particular, *Begriffe* are *Bedeutungen*, though for purpose of the argument it makes no substantial difference. I shall, in effect, argue that, however Frege would have answered, Functions in fact are neither *Sinne* nor *Bedeutungen*, but are rather, so to speak, ranges of language acts.

⁸ Frege did, in the other hand, hold that Functions can be arguments of (higher order) Functions, and it is partly, I suspect, his demand that Functions resemble Objects in this *one* respect which left him prey to the puzzle just set forth.

predicates or something comparable. A predicate is a piece of notational machinery employed to make or spoken of in connection with making statements of a certain sort; if we employ the predicate under appropriate conditions, then we do make a statement of that sort. To employ a predicate is to put a name or other referring expression into one of the kinds of sentence contexts where it does assume reference. Indeed, rightly regarded, Frege's point is simple truth.

Difficulty comes not from the evasive physiognomy of Concepts, or from the ineptness of predicates, but from Frege's conception of a Concept. We are puzzled because we carry in the back of our minds the idea that a Concept -- the alleged metaphysical counterpart of a predicate -- must be some queer kind of object. But what the argument shows is that a Concept is no kind of object at all. Frege thought the Concept was something like an Object because he thought that the distinction between referring and predicating must be echoed by a metaphysical distinction; and, as remarked, metaphysical distinctions can be drawn only between objects of metaphysically distinct kinds. Frege was led into this mistake by rightly noting that a meaningful word or combination of words is more than a mark, it also has a meaning or indicates a connection of meanings. He then wrongly concluded that this meaning or indicated connection of meanings is a *Bedeutung* which the expression or complex of expressions "means". Now apart from whether all uses are referring uses, we must carefully distinguish the connection of meanings indicated by a predicate from the meaning which the various simple expressions in it might have. In our terms, the predicate itself has no use (of words), though the expressions embodied in it do. What it indicates is a range of complete language acts having features in common, and which differ in some one definite respect as is indicated by the ways in which we can fill in the predicate to get a proper sentence.

Frege wanted it both ways. In one mind he thought that all combinations of meaningful signs must work much the same; but in another mind, he saw important differences between the uses of referring expressions and predicates. We might say that Frege

went wrong and made the Concept into a queer thing, somehow like yet so unlike an Object, because he put predicating on the same level with referring; he made them somehow equal partners in the truth-value; to put it otherwise, he made referring expressions and predicates assume equal and simply complementary roles in the sentence. In fact, the predicate, if it *may* be said to have a use,⁹ is used to predicate, i.e., to make a statement of a certain kind. Frege tried to compare two things which are not directly comparable: a use of words and a type of language act.

Concluding Note. It may be objected that this cannot be the correct analysis of Frege's difficulty, for he most certainly did allow that different "propositional attitudes", such as questioning and asserting, might attach to the same *Gedanke*, as expressed by a sentence built of a referring expression and a predicate. But all this assumes what is highly questionable, namely the correctness of Frege's "identical content" analysis of the relations between questions, statements, etc. In any case, while Frege does tell us what the *Bedeutung* of a statement-making sentence is, he never reveals what the *Bedeutung* of (say) a question is to be, and so we never know what is to be the *Bedeutung* of a predicate used to ask a question. If Frege were to reply as I supposed he might, then I would strongly suspect that here again he is confusing distinctions by supposing them to be independent. I mean: Frege seemed to think the three kinds of distinction he made between *Sinn* and *Bedeutung*, *Gegenstand* and *Funktion*, and assertion and other "propositional attitudes" (uses of Language) independent in the sense that we can go down the list: first determine the *Sinn* of the expression; then determine whether its *Bedeutung* is an Object or a Function; then determine whether the expression is being used to assert, question, etc. or perhaps contributes to a mere *beurteilbare Inhalt*. Examination shows, in fact, that these distinctions cannot be independent in this way. It is therefore no accident that Frege never succeeds in explaining the *Sinn* of expressions other than sentences and (perhaps) de-

⁹ It is less confusing to speak about the use, not of the predicate, but of the occurring general-names, copulas, etc.

scriptive phrases, that in the *Grundgesetze* he places the "Thought" as the *Sinn* of sentence (v.I., # 32), while in his last papers,¹⁰ reverting to an idea intimated in the *Begriffsschrift*, he is inclined to cast the Thought as the complete "content" of a judgement and, finally, he explains the *Bedeutung* of words only as they occur in *statement-making* uses of language. Not even Frege's celebrated rigor of mind sufficed to eliminate the uncertainty and incoherence which inevitably come from trying to use too simple a system.

¹⁰ See "The Thought: A Logical Inquiry" (tr. by A. M. and Marcelle Quinton), *Mind*, 1956, e.g. p. 294.

On minimal models of first-order systems

by

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1. *Introduction.* The theorems presented below strengthen some results stated in my previous paper (2). One of the main results of (2) was the stating of a sufficient condition for a complete consistent system (L, Σ) to have a model of finite character. The condition can be formulated: the number of maximal consistent condition-sets in (L, Σ) is at most denumerable. (T 8 in (2).) We will here show that a both necessary and sufficient condition for the existence of a model of finite character of (L, Σ) is that all the Boolean algebras $B(\Sigma, a_1, \dots, a_r)$ are atomic (Thms 3-4). We observed in (2) that a model of finite character has a certain minimality property (T 5). We will show here also that only models of finite character can have this minimality property (Thm 2). Consequently, we may identify minimal models with models of finite character.

We will use, with a few exceptions, the same notation and terminology as in (2). A system is thought of as an ordered couple (L, Σ) where L is a language in first-order calculus with identity and Σ is a set of axioms. A symbol-sequence

$$\Phi_1, \Phi_2, \dots \rightarrow \Psi_1, \Psi_2, \dots$$

where the $\Phi_1, \Phi_2, \dots, \Psi_1, \Psi_2, \dots$ denote either formulas or sets of formulas, is used to state that every formula on the right-hand side is deducible (with all free variables kept constant) from formulas on the left-hand side. If L is a language and a_1, a_2, \dots, a_r are individual constants not belonging to L , we denote by $L(a_1, a_2, \dots, a_r)$ the language formed by adjoining the individual symbols a_1, a_2, \dots, a_r to L .

2. Construction of a model of a system (L, Σ)

We will here describe a general procedure for constructing models, which we will use later on. Our procedure differs only slightly from that of Henkin in (1), but seems to be handier for our purposes.

We let (L, Σ) be a consistent system. We let $a_{11}, a_{12}, \dots; a_{21}, a_{22}, \dots; \dots; a_{k1}, a_{k2}, \dots; \dots$ be individual constants not belonging to L , and we designate by K_n the class consisting of those constants a_{ij} for which $i \leq n$ and $j \leq n$. We let L_n be the language obtained by adjoining to L the constants of K_n , and we let L_ω be the language obtained by adjoining all the symbols a_{ij} to L . We now will extend the system (L, Σ) to a complete consistent system $(L_\omega, \Sigma_\omega)$ which will "describe" a model M of (L, Σ) . We form this system $(L_\omega, \Sigma_\omega)$ by the following stepwise construction.

0a) We fix an enumeration S of all the formulas in L_ω .

0b) We extend (L, Σ) to a maximal consistent system (L, Σ_0) .

1a) We let $(\exists x)A_{11}(x), (\exists x)A_{12}(x), \dots$ be all the theorems of (L, Σ_0) of the form $(\exists x)\Phi(x)$, in the order of their appearance in the enumeration S . We form the set Σ'_0 by adding the sentence $A_{11}(a_{11})$ to Σ_0 .

1b) We form (L_1, Σ_1) by extending (L_1, Σ'_0) to a maximal consistent system.

Generally, for $n \geq 1$: Assuming that the system (L_{n-1}, Σ_{n-1}) has already been constructed, we form (L_n, Σ_n) in the following two steps.

na) We designate by $(\exists x)A_{ij}(x)$ the j :th (in the enumeration S) theorem of Σ_{i-1} of the form $(\exists x)\Phi$. We form the system Σ'_{n-1} by adding to Σ_{n-1} the sentences $A_{ij}(a_{ij})$ for $i=n, j \leq n$, and for $j=n, i \leq n$.

nb) We form the system (L_n, Σ_n) by extending (L_{n-1}, Σ'_{n-1}) to a maximal consistent system.

The system Σ_ω is now defined as the sum of all the systems Σ_n for $n=1, 2, \dots$. It can easily be proved, in the same way as in Henkin (1), that the system Σ_ω can be regarded as describing a model M of (L, Σ) , all the elements of which have some of the constants a_{ij} as a name.

3. On condition-sets which need not be satisfied

Def. If $\{B_i(x_1, \dots, x_s)\}$ is a set of formulas of L containing only the variables x_1, \dots, x_s free we call $\{B_i(x_1, \dots, x_s)\}$ a *condition-set* in x_1, \dots, x_s . We say that a condition-set $\{B_i(x_1, \dots, x_s)\}$ has a *finite basis*, or simply, *has a basis*, in (L, Σ) , if there exists a formula $R(x_1, \dots, x_s)$ in L such that $\Sigma \rightarrow (\exists x_1) \dots (\exists x_s) R(x_1, \dots, x_s)$ and such that

$$\Sigma, R(x_1, \dots, x_s) \rightarrow \{B_i(x_1, \dots, x_s)\}.$$

If this holds, we can see, that if Σ is satisfied in a structure M , then $\{B_i(x_1, \dots, x_s)\}$ must also be satisfied in M . We also observe that if there are constants a_1, \dots, a_s such that $\{B_i(a_1, \dots, a_s)\}$ follows from Σ , then according to the definition $\{B_i(x_1, \dots, x_s)\}$ has a basis in (L, Σ) . For $R(x_1, \dots, x_s)$ we may here take the formula

$$x_1 = a_1 \cdot x_2 = a_2 \cdot \dots \cdot x_s = a_s.$$

We want to prove the following theorem.

Theorem 1. Let (L, Σ) be a complete consistent system, and let $\Gamma = \{B_i(x_1, \dots, x_s)\}$ be a condition-set in x_1, \dots, x_s which is without finite basis in (L, Σ) . Then it is possible to find a model M of (L, Σ) , in which the condition-set Γ is not satisfied.

Before proving the theorem we prove two simpler lemmas.

Lemma 1. Let (L, Σ) be a complete consistent system, let $\{B_i(x_1, \dots, x_s)\}$ be a condition-set without finite basis in (L, Σ) , let L' be the language $L(a_1, \dots, a_r)$ formed by adjoining the individual constants a_1, \dots, a_r to L , and let $A(a_1, \dots, a_r)$ be a sentence of L' which is consistent with Σ . Then, if $R(a_1, \dots, a_r, x_1, \dots, x_s)$ is any formula of L' which is consistent with $\Sigma \cup \{A(a_1, \dots, a_r)\}$ and which contains no variables free other than x_1, \dots, x_s , there is an index i such that the sentence $(\exists x_1) \dots (\exists x_s) (R(a_1, \dots, a_r, x_1, \dots, x_s) \cdot \sim B_i(x_1, \dots, x_s))$ is consistent with $\Sigma \cup \{A(a_1, \dots, a_r)\}$.

Proof: We assume that the assertion of the lemma does not hold. This means that for every i the sentence

$$(x_1) \dots (x_s) (R(a_1, \dots, a_r, x_1, \dots, x_s) \supset B_i(x_1, \dots, x_s))$$

is derivable from $\Sigma \cup \{A(a_1, \dots, a_r)\}$. This can also be expressed by the formula

$$\Sigma, A(a_1, \dots, a_r), R(a_1, \dots, a_r, x_1, \dots, x_s) \rightarrow \{B_i(x_1, \dots, x_s)\}.$$

From this we can deduce

$$\Sigma, (\exists y_1) \dots (\exists y_r) (A(y_1, \dots, y_r) \cdot R(y_1, \dots, y_r, x_1, \dots, x_s)) \rightarrow \{B_i(x_1, \dots, x_s)\}.$$

However, since the left-hand side in this relation is consistent, and the existential statement is a formula in L , it follows that $\{B_i(x_1, \dots, x_s)\}$ has a finite basis in (L, Σ) . This is contrary to our assumption. This completes our indirect proof of the lemma.

Lemma 2. Let (L, Σ) be a complete consistent system, let $\Gamma = \{B_i(x_1, \dots, x_s)\}$ be a condition-set without finite basis in (L, Σ) , let L' be the language $L(a_1, \dots, a_r)$ formed by adjoining the individual constants a_1, \dots, a_r to L , and let A be a sentence of L' which is consistent with Σ . Then it is possible to find a complete consistent system (L', Σ') such that Σ' includes Σ and A , and such that Γ does not have a finite basis in (L', Σ') .

Proof. It is evidently sufficient if we construct the set Σ' so that it contains Σ and A , is maximal consistent in L' , and satisfies the following condition:

(1) For every formula $R(x_1, \dots, x_s)$ in L' which is consistent with Σ' and which does not contain any variables free other than x_1, \dots, x_s , there is an index i such that Σ' contains the sentence $(\exists x_1) \dots (\exists x_s) (R(x_1, \dots, x_s) \cdot \sim B_i(x_1, \dots, x_s))$.

We proceed to the construction of such a set Σ' . We first make an enumeration $S: R_1(x_1, \dots, x_s), R_2(x_1, \dots, x_s), \dots$ of all the formulas of L' which contain only the variables x_1, \dots, x_s free. Then we define a sequence A_0, A_1, A_2, \dots of successive extensions of $\Sigma \cup \{A\}$ by the following inductive procedure:

$$A_0 \text{ is defined as } \Sigma \cup \{A\}.$$

For $n > 0$, A_n is defined by the conditions: If $R_n(x_1, \dots, x_s)$ is not consistent with A_{n-1} , then $A_n = A_{n-1}$; and if $R_n(x_1, \dots, x_s)$ is consistent with A_{n-1} , then A_n is formed from A_{n-1} by addition of the sentence $(\exists x_1) \dots (\exists x_s) (R_n(x_1, \dots, x_s) \cdot \sim B_t(x_1, \dots, x_s))$, where t is the smallest index which makes this sentence consistent with A_{n-1} . (By lemma 1 we know that there exists some such index.)

Since each of the sets A_n is consistent, their sum A is also consistent, and we can construct Σ' as a maximal consistent extension in L' of A . It is easy to see that this set Σ' satisfies condition (1) above. Assume that $R(x_1, \dots, x_s)$ is any formula of L' , which contains only the variables x_1, \dots, x_s free and which is consistent with L' . This formula must occur somewhere in the enumeration S , i.e. for some k it is identical with the formula $R_k(x_1, \dots, x_s)$. Since by assumption the formula $R(x_1, \dots, x_s)$ is consistent with Σ' , it is also consistent with A_{n-1} ; and therefore, by the rule of construction for A_n , for some index i the sentence $(\exists x_1) \dots (\exists x_s) (R(x_1, \dots, x_s) \cdot \sim B_i(x_1, \dots, x_s))$ is in A_n , and therefore also in Σ' . This shows that the system (L', Σ') fulfills condition (1), and therefore that the condition-set $\{B_i(x_1, \dots, x_s)\}$ lacks a finite basis in (L', Σ') .

Proof of Theorem 1. Combining the lemma just proved with the method for constructing models described in section 2, it is now easy to prove theorem 1. The lemma shows that it is possible to perform the completion processes (steps 1b, 2b, ... in the model-construction) so that Γ lacks a finite basis in Σ_n , if it lacks a finite basis in Σ_{n-1} . Therefore, under the assumptions of the theorem, we can form the system Σ_ω in such a way that $\{B_i(a_1, \dots, a_s)\}$ is not contained in Σ_ω for any choice of constants a_{ij} for a_1, \dots, a_s . Clearly, the condition-set Γ is not satisfied in the model corresponding to this system Σ_ω . This concludes the proof of the theorem.

We can also prove the following generalization of theorem 1.

Theorem 1'. Let (L, Σ) be a complete consistent system, and let $\Gamma_1, \Gamma_2, \dots$ be a sequence of condition-sets, each of which is without finite basis in (L, Σ) . Then it is possible to find a model M of (L, Σ) , in which none of the condition-sets $\Gamma_1, \Gamma_2, \dots$ is satisfied.

This theorem can easily be seen to follow from the following generalization of lemma 2.

Lemma 2'. Let (L, Σ) , L' , and A be as in the assumption of lemma 2, and let $\Gamma_1, \Gamma_2, \dots$ be a sequence of condition-sets, none of which has a finite basis in (L, Σ) . Then it is possible to find a complete consistent system (L', Σ') such that Σ' includes Σ and

A , and such that none of the condition-sets $\Gamma_1, \Gamma_2, \dots$ has a finite basis in (L', Σ') .

Sketch of proof. The only change which must be made in the proof of lemma 2 is a modification of the definitions of the sets A_1, A_2, \dots . We now define these sets as follows:

We first make an enumeration $S: p_1, p_2, \dots$ of all the pairs (Γ, R) , where Γ is one of the condition-sets $\Gamma_1, \Gamma_2, \dots$ and R is a formula of L' , which contains no variables free other than those of Γ .

We define A_0 as the set $\Sigma \cup \{A\}$.

When $n > 0$, we may assume that A_{n-1} is already constructed, and we may let p_n be the pair consisting of the condition-set $\{B_i(x_1, \dots, x_s)\}$, and the formula $R(x_1, \dots, x_s)$. If $R(x_1, \dots, x_s)$ is not consistent with A_{n-1} , we let A_n be identical with A_{n-1} ; and if $R(x_1, \dots, x_s)$ is consistent with A_{n-1} , we form A_n from A_{n-1} by adding the sentence $(\exists x_1) \dots (\exists x_s) (R(x_1, \dots, x_s) \cdot \sim B_t(x_1, \dots, x_s))$, where t is the smallest index which makes this sentence consistent with A_{n-1} . (Also in this case, the existence of such an index t is guaranteed by lemma 1, since A_{n-1} is consistent and formed from Σ by addition of a finite number of sentences in L' .)

When Σ' is taken as a maximal consistent system in L' which includes all the systems A_n , it is seen in the same way as in the proof of lemma 2, that none of the condition-sets $\Gamma_1, \Gamma_2, \dots$ can have a finite basis in (L', Σ') .

We will now use theorem 1 to prove a theorem about minimal models. If (L, Σ) is a complete consistent system, we mean by a *minimal model* of (L, Σ) a model M such that every other model of (L, Σ) is isomorphic with an arithmetical extension of M . We know ((2) T5) that if M is a model of finite character, then it is a minimal model in this sense.¹ We now prove the converse.

Theorem 2. If (L, Σ) is a complete consistent system, and M

¹ A model M of a complete consistent system (L, Σ) is said to be of *finite character*, if every maximal consistent condition-set $\Gamma(x_1, \dots, x_r)$, which is satisfied in M , has a finite basis in (L, Σ) ; i.e. if there is one condition $B(x_1, \dots, x_r) \in \Gamma(x_1, \dots, x_r)$ such that $\Sigma, B(x_1, \dots, x_r) \rightarrow \Gamma(x_1, \dots, x_r)$. Cf. (2) p. 85.

is a model of (L, Σ) , then M can be minimal only if it is a model of finite character.

Proof: Let M be a model of (L, Σ) , which is not of finite character. This means, that some condition-set $\{B_i(x_1, \dots, x_s)\}$ which lacks a finite basis in (L, Σ) is satisfied in M . But then there is, according to theorem 1, some other model M' where $\{B_i(x_1, \dots, x_s)\}$ is not satisfied. This model M' can not be an arithmetical extension of M . It follows, that M is not a minimal model. This concludes the proof of the theorem.

4. *A condition for existence of models of finite character (minimal models)*

If (L, Σ) is a system and a_1, \dots, a_r are individual constants not belonging to L , the sentences of $L(a_1, \dots, a_r)$ form a certain Boolean algebra which we designate $B(\Sigma, a_1, \dots, a_r)$, where the Boolean relation " A is included in B " is interpreted as

$$\Sigma, A \rightarrow B$$

A sentence A of $L(a_1, \dots, a_r)$ is an *atom* in this algebra if A is consistent with Σ (i.e. different from the null element of the algebra) and if for every other sentence B of $L(a_1, \dots, a_r)$, if B is consistent with Σ , and $\Sigma, B \rightarrow A$, then $\Sigma, A \rightarrow B$.

The algebra $B(\Sigma, a_1, \dots, a_r)$ is called *atomic*, if for every sentence A of $L(a_1, \dots, a_r)$ which is consistent with Σ there exists a sentence B such that B is an atom of the algebra, and such that $\Sigma, B \rightarrow A$.

We will be concerned in particular with maximal consistent deductive systems in $L(a_1, \dots, a_r)$ which contain Σ . These systems are the maximal dual ideals in the algebra $B(\Sigma, a_1, \dots, a_r)$. If A is an atom in $B(\Sigma, a_1, \dots, a_r)$, then the dual ideal generated by A is maximal. And conversely, if the dual ideal which is generated by an element A is maximal, then A is an atom in $B(\Sigma, a_1, \dots, a_r)$. Translating to the logical language, we get the following lemma.

Lemma 3. If A is a sentence of $L(a_1, \dots, a_r)$, then $\Sigma \cup \{A\}$ determines a complete consistent system in $L(a_1, \dots, a_r)$ if and only if A is an atom in the algebra $B(\Sigma, a_1, \dots, a_r)$.

Using this lemma and the model-construction of section 2, we can now prove the following theorem.

Theorem 3. If (L, Σ) is a complete consistent system, and for every finite set $\{a_1, \dots, a_r\}$ of individual constants not belonging to L the algebra $B(\Sigma, a_1, \dots, a_r)$ is atomic, then the system (L, Σ) has a model M of finite character.

Proof. In constructing the model, we follow the general procedure described in section 2. We want to show that the systems Σ_n can be chosen so that for every n there exists a sentence C_n in $L(a_{11}, \dots, a_{nn})$ such that Σ_n is determined by the axiom system $\Sigma' \cup \{C_n\}$. We can see that this holds for $n=1$. After forming Σ'_0 by adding $A_{11}(a_{11})$ to Σ_0 we can form the complete extension Σ'_1 by adding one single axiom $C_1(a_{11})$, where $C_1(a_{11})$ is an atom of $B(\Sigma', a_{11})$ "contained in" $A_{11}(a_{11})$ (i.e. such that $\Sigma', C_1(a_{11}) \rightarrow \rightarrow A_{11}(a_{11})$). The completeness of $\Sigma' \cup \{C_1\}$ is guaranteed by lemma 3. We show next that the assertion holds for n , if it holds for $n-1$. Assume that the complete system Σ_{n-1} is determined by the axiom system Σ, C_{n-1} . The system Σ'_{n-1} will be Σ, C_{n-1} together with a finite number of sentences $A_{ij}(a_{ij})$, where $a_{ij} \in K_n$. We can now form the complete extension Σ'_n by adding as an axiom an atom C_n of $B(\Sigma, a_{11}, \dots, a_{nn})$ "included" in the conjunction of the sentence C_{n-1} and the new sentences $A_{ij}(a_{ij})$. Again we know by lemma 3 that the axiom system Σ, C_n is complete.

It is easy to see that the model M resulting from this construction is of finite character. Let $\{A_i(a_1, \dots, a_s)\}$ be the set of all theorems of Σ_ω which contain only the individual constants a_1, \dots, a_s . This set must be contained in Σ_n for some n . Therefore it holds that

$$\Sigma, C_n(a_1, \dots, a_s, \beta_1, \dots, \beta_t) \rightarrow \{A_i(a_1, \dots, a_s)\}$$

where the constants $a_1, \dots, a_s, \beta_1, \dots, \beta_t$ together constitute the set a_{11}, \dots, a_{nn} of the individual constants of C_n . Therefore it also holds that

$\Sigma, (\exists x_1) \dots (\exists x_t) C_n(a_1, \dots, a_s, x_1, \dots, x_t) \rightarrow \{A_i(a_1, \dots, a_s)\}$. This shows that the condition-set $\{A_i(a_1, \dots, a_s)\}$ has a finite basis in (L, Σ) . Since this holds for an arbitrary sequence a_1, \dots, a_s , we conclude that every condition-set which is satisfied in M

has a finite basis in (L, Σ) , i.e. that M is a model of finite character.

The following corollary is identical with theorem 8 of (2). (The method used here to arrive at this result seems essentially simpler than that used in (2).)

Corollary. If (L, Σ) is a complete consistent system, and for every finite set $\{a_1, \dots, a_r\}$ of constants not belonging to L the number of maximal dual ideals in the algebra $B(\Sigma, a_1, \dots, a_r)$ is at most denumerable, then (L, Σ) has some model M of finite character.

The corollary follows from the theorem via the observation, that a non-atomic Boolean algebra must have a more than denumerable number of maximal dual ideals.

The converse of theorem 3 is also true.

Theorem 4. If (L, Σ) is a complete and consistent system which has a model of finite character, then for any finite set of constants a_1, \dots, a_r not belonging to L the algebra $B(\Sigma, a_1, \dots, a_r)$ is atomic.

Proof. Let $A(a_1, \dots, a_r)$ be an arbitrary sentence of $L(a_1, \dots, a_r)$, which is consistent with Σ , and let M be a model of (L, Σ) of finite character. Since $(\exists x_1) \dots (\exists x_r) A(x_1, \dots, x_r)$ is a theorem of (L, Σ) , there is some sequence a_1, \dots, a_r of elements in M which satisfies the condition $A(x_1, \dots, x_r)$. Since M is of finite character, there is some condition $B(x_1, \dots, x_r)$ which is satisfied by a_1, \dots, a_r , and such that $\Sigma, B \rightarrow C$ holds for every other condition C which is satisfied by a_1, \dots, a_r . This means that the set $\Sigma \cup \{B(a_1, \dots, a_r)\}$ determines a complete consistent system in $L(a_1, \dots, a_r)$. Using lemma 3 we deduce that $B(a_1, \dots, a_r)$ is an atom contained in $A(a_1, \dots, a_r)$. We have thus proved that an arbitrary non-null element of $B(\Sigma, a_1, \dots, a_r)$ contains some atom, i.e. that the algebra $B(\Sigma, a_1, \dots, a_r)$ is atomic.

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A Study in Hobbes' Theory of Denotation and Truth

by

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Although Thomas Hobbes deserves a good reputation as a semanticist, very few studies of his semantics *from a modern point of view* have been carried out. A noticeable exception is a paper entitled "On the Semantics of Thomas Hobbes" by R. M. Martin in *Philosophy and Phenomenological Research* Vol. XIV, no. 2, 1953.

It is perhaps no accident that Martin writes about Hobbes: Martin shares Hobbes' nominalistic inclinations and in particular Hobbes' idea of multiple denotations. He has carried out what is essentially a program in Hobbes' own publications.¹

In this study I wish to examine Hobbes' semantic ideas somewhat more closely than Martin did in his paper. The best tribute to Hobbes is to connect the pertinent passages in his writings and to present his semantics in a more closed form. This task requires some reconstruction.

I. *Theory of Denotation*

One fundamental notion in Hobbes' semantics is that of *name*. Hobbes has not defined this notion properly. It is true that there is a passage which is labelled "definition of name" in Hobbes' main semantic work "Computation or Logic". The passage is the following:²

¹ R. M. Martin and J. H. Woodger, "Toward an Inscriptional Semantics," *Journal of Symbolic Logic*, XVI (1951), 191-203.

² Thomas Hobbes, *English Works*, ed. by Molesworth, 1839, Vol. 1, Part. 1, Sect. 4. (Abbreviated: E. W., I. 1. 4.)

"A *name* is a word taken at pleasure to serve for a mark, which may raise in our mind a thought like to some thought we had before, and which, being pronounced to others, may be to them a sign of what thought the speaker had, or had not in his mind."

If this passage is taken to be a definition of name, we must declare that it has a low degree of adequacy:

A. Names are said to be single words; but Hobbes explicitly states in many passages that there exist composite names consisting of more than one word, and he gives many examples of such names, e.g., "He who wrote the Iliad".

B. It is not recommendable to define names in terms of mental occurrences associated with expressions, because

a) It is difficult to recognize what is and what is not a name from such mental occurrences;

b) Major parts of a psychological account of what normally takes place in communication will acquire analytic character instead of being what they should be, viz., empirical hypotheses to be confirmed by observations.

The passage above will gain in adequacy if it be relabelled: "Hypotheses as to what takes place in communication". We should ask whether the passage is true or not.³

As a proper definition of "name" is lacking, I wish to provide one which will be recommended by its agreement with what Hobbes writes about names and with his examples of what he will call "names" and what he will not call "names".

The definition of "name" I will propose is the following:

Definition of "Name", (D. N.):

An expression E is a *name* =_{def} E can be placed before "is" (or "are") and/or can be placed after "is (or "are") in the sentence form " - - - (^{is} are) - - - " so as to yield a significant sentence.

³ Personally I doubt that a listener normally comes to think on what may occur in the mind of a speaker. Only if he does so, is it true to say that names are signs for the listener of what occurs in the mind of the speaker. However it happens frequently that a listener receives a speech as symptoms of mental processes in the mind of the speaker; and that is what Hobbes actually states.

Hobbes has not explicitly stated anything like this definition in his works. Nevertheless I think that it has a high degree of adequacy by *the criterion of agreement with Hobbes' usage*.

a) All his examples of names are names according to D. N.

b) He gives also examples of expressions which he will not call names:⁴ "But these words, 'all', 'every', 'some', etc., ... are not names but parts only of names ...". These expressions are not names according to D. N.

c) The best support for my contention that the definition is a good one by *the criterion of agreement* is perhaps this passage:⁵ "A proposition is a speech consisting of two names copulated, by which he that speaketh signifies he conceives the latter name to be the name of the same thing whereof the former is the name; or ... that the former name is comprehended by the latter."

Should we judge the definition by another criterion of adequacy which would be appropriate in a systematic semantic study, namely *the criterion of fertility or suitability*: A term introduced by a definition ought to be useful, then I think the verdict would be less favorable:

The sign of negation is treated as part of a name and not as a connective. The words "all", "every", "each", and "some" are also treated as parts of names and not as quantifiers. The advantages of the modern way of handling these expressions are telling against the advantage of defining names as in D. N.

Hobbes divides names into *universal*, *particular*, and *individual* ones.⁶ His "definitions" have a mixed character; he refers to the form of expressions, and at the same time he describes the semantic properties of these kinds of name in definition-like passages.

It is a good policy to attempt to define kinds of expressions in such a way that the definitions are as little committal as possible concerning the semantical aspects of the expressions.

Examples of universal names are "all men", "all philosophers", "every politician", etc., and of particular names "some men",

⁴ E. W., I. 2. 11.

⁵ E. W., I. 3. 2.

⁶ E. W., I. 2. 11.

"some philosophers", "some politicians", etc. The following expressions exemplify individual names, "Homer", "He who wrote the Iliad", etc.

Can we define "universal names", "particular names", and "individual names" from the form of expressions? The examples given seem to indicate that this is possible. However, a closer examination shows that there are expressions the form of which does not tell us to what class they belong. Compare the sentences

"The lion is five years old" (S₁)

and

"The lion is a mammal" (S₂)

The occurrence of "The lion" in S₁ is an individual name, whereas its occurrence in S₂ is a universal name. This kind of ambiguity is of importance and urges us to make a distinction well-known in modern semantics, namely the distinction between *type* and *token*. When we say that the first name in S₁ is the same as (or identical with) the first name in S₂, then we are speaking about a type. When we say that the expressions are different occurrences, then we are speaking about tokens.

The ambiguity referred to above can be more precisely stated in this form: A name-type has an ambiguous status with regard to the classification into universal, particular, and individual names if the type has tokens which belong to different classes.

Hobbes' classification into universal, particular, and individual names is intended to be a logical division. We have just seen that it is no such division if the domain of the classification is types of names. We conclude that it ought to be regarded as a classification of tokens.

It is my objective first to explicate Hobbes' classification of name-tokens into universal, particular, and individual ones, and then to discuss pertinent semantic properties of them. The plan for the explication will be as follows:

a) To look for tokens, the types of which do not have ambiguous status in the classification.

b) To place other tokens into the same classes as tokens of the former kind, if they can be replaced by them in the contexts in which they appear, without change of meaning.

The notion of *contextual synonymy* is important. Let us schematize the planned definition.

a) Tokens of the form so-and-so belong to the class so-and-so.

b) If token t in the context $---t---$ be replaced by the token t' , and if $---t---$ and $---t'---$ are synonymous, then t and t' belong to the same class.

Our definitions can now be stated:

Definition of universal name, D. U.:

A token t is a *universal* name-token =_{def} t has the form "all ---s", or t is contextually synonymous with a token having this form.

Definition of particular name, D. P.:

A token t is a *particular* name-token =_{def} t has the form "some ---s", or t is contextually synonymous with a token of this form.

Definition of individual name, D. I.:

A token t is an individual name (or a singular name) =_{def} t is a proper name, or t is contextually synonymous with a proper name.

Definition of contextual synonymy, D. C. S.:

A token t in the context $---t---$ is synonymous to a token t' , if the result $---t'---$ of replacing t in $---t---$ by t' has the same meaning as $---t---$.

These definitions are, I believe, rather good if they are judged by the criterion of agreement: Hobbes' examples of universal, particular, and individual names satisfy the definitions. If, however, they are judged by the criterion of fertility, what was said about D. N. above applies, *mutatis mutandis*, to the new definitions also.

We shall now examine the denotational properties of the various kinds of name tokens defined above.

1. *Universal name-tokens.* Hobbes declares that these name-tokens have *certain* or *determined* denotations, and that they

denote *distributively* every member of a set of objects. "All men" for example denotes every individual man.

Hobbes has not observed that there are different kinds of universal names having different denotational properties. Compare the first name-tokens in the following sentences:

"All men are mortal." (S_3)

"All the horses are in the stable." (S_4)

"All the apostles are twelve persons." (S_5)

"All men" in S_3 denotes distributively every individual which is a man. "All the horses" in S_4 may be said to denote distributively but may also be said to denote a collection of horses (for instance, the horses of Mr. Jones, if a *wider context* makes this clear). A marked difference between this token and the token "all men" in S_3 is that it has a restricted universality. Furthermore, it is not clear *from the sentence S_4 alone* to what population of horses the token "all the horses" has reference. We can therefore say that another difference between the two tokens under discussion is that whereas the first name-token in S_3 has determined denotation, this is not true for the first name-token in S_4 . The first name-token in S_5 finally has definite denotation and denotes a collection.

We find that universal name-tokens should be subdivided into three logical divisions, viz. into:

- | | | |
|----------------|------------------|------------------|
| a) 1. Definite | b) 1. Restricted | c) 1. Collective |
| and | and | and |
| 2. Indefinite | 2. Unrestricted | 2. Distributive |

Hobbes has only discerned universal name-tokens which belong to the class-product $a_1 \cdot b_2 \cdot c_2$.

It is worth noting that universal name-tokens belonging to b_1 normally also belong to a_2 . They are *semantical'y open* if they do not occur in a wider discourse context than a single sentence detached from a pragmatic situation.

A criterion by means of which one may find out whether a given universal name-token is distributive is this:

Definition of distributive universal name-tokens, D. D. U.:

t is a distributive universal name-token $=_{\text{def}}$ t is a token of the form "every —", or is contextually synonymous with such a token.

2. *Particular name-tokens.* Hobbes declares that particular name-tokens have the form "some —". Other tokens which ought to be classified as particular names are tokens of the form "most —", "80 % of —", etc. Tokens of the form "—s" may occasionally be particular. An example is the first name in the following sentence:

"Dogs dislike cats." (S₆)

For S₆ has the same meaning as

"Almost all dogs dislike cats." (S₇)

The token "Dogs" in S₆ belongs therefore to the same class as the token "Almost all dogs" in S₇, i.e., it is a particular name-token.

Hobbes' account of the denotational properties of particular names should be viewed in the light of his theory of truth which will be discussed more fully later on in this paper.

Hobbes' definition of truth can be stated as follows:

"A proposition of the form α e β , where α and β are names and e a copula, is true if β denotes every object which is denoted by α ."

Hobbes' contention that expressions of the form "some —" are names, together with his theory of truth, have consequences which are not satisfactory. Let us consider the following true sentence:

"Some Greeks are very rich." (S₈)

What does it mean to say that S₈ is true? Hobbes' answer would be: It is true because the second name denotes every object which is denoted by the first name, i.e., the name Very rich denotes every member belonging to a set of Greek persons. But what set? How do we know what objects belong to it, and what objects do not belong to it? Without such knowledge we would be at a loss when we try to use Hobbes' criterion of truth.

It would not do to declare that the set of Greeks is the set of very rich Greeks, for in that case it would hold that S_8 is synonymous to:

"Some very rich Greeks are very rich." (S_9)

S_8 and S_9 are obviously not synonymous. There is no alternative but to declare that the set of Greeks, every member of which is denoted by the first name-token in S_8 , is indefinite. This is Hobbes' contention.

Let us consider another true sentence with a particular first name-token, viz.:

"Some Greeks are very poor." (S_{10})

Hobbes would say that S_{10} is true because the token "Some Greeks" denotes every member of *a set of Greek persons* which is also denoted by the name "Very poor". Once again: What set of Greeks? It is evident that this set is different from the former set.

Thus, not only are the sets of denoted objects for particular name-tokens indefinite in the sense that the question is left open which objects do and which objects do not belong to them, but they may also be different for different tokens of the same type.

It is informative to contrast a modern answer to the question: "In what sense is S_8 true?" to Hobbes' answer. Modern philosophers would say that S_8 has the logical form:

$(\exists x)(x \varepsilon \text{ Very rich Greeks})$ (S'_8)

This sentence contains only *one* name, viz., the name Very rich Greeks, which name denotes a class. The sign x just before the sign ε is not a name but a variable. It does not have denotation, but it has a range of values. (S'_8) is true if the formula " $x \varepsilon \text{ Very rich Greeks}$ " is true for at least one value in the range of values of the variable x .

We notice that Hobbes is committed to operate with indefinite sets because he regards "some" as part of a name. If we adopt the modern alternative of turning the logical particle "some" into a quantifier, we avoid this inconvenience.

3. *Individual name-tokens.* Hobbes' examples of individual

name-tokens include proper names, "Homer", and definite descriptions, "He who wrote the Iliad". We have earlier argued that it is necessary to distinguish between tokens and types, if we want to establish the status of a name as universal, particular, or individual. If it be said that a description denotes a definite individual, we must keep the same distinction in mind. Let us consider a number of sentence-tokens of the same type as the following one:

"That tree is an oak."

(S₁₁)

A first sentence-token is uttered, let us suppose, by a person pointing to an oak, a second one by a person pointing to a palm-tree. The first name-tokens in these two sentence-tokens do obviously denote different objects.

It may happen that an individual name-token fails to denote any object at all. For example, the first name in a sentence-token of the same type as S₁₁ uttered by a person in the middle of a tree-less desert. Such name-tokens may be called *vacuous*. We shall see that vacuous name-tokens present difficulties for Hobbes' theory of truth. Hobbes is to some extent aware of the problems.

The first name-token in a sentence-token of the same type as S₁₁ may have "improper" denotational properties also in another way. Suppose that a token of the same type as S₁₁ is uttered by a person pointing to a wood from such a distance that it is impossible to discern individual trees. We have here a case of *embarras de richesse*. Hobbes has not himself called attention to this kind of improper denotation, but it is relevant to a discussion of his theory of truth.

Not only individual name-tokens may exhibit anomalies with respect to denotation. There are also vacuous universal name-tokens. In fact all tokens of the same type as "all Olympian gods" are vacuous. The fact that there are denotational anomalies is a good reason to proceed as we have done above, viz., to define kinds of names without mentioning their denotational properties in the definitions. Suppose that we had done otherwise and defined, for example, individual names as names denoting

individuals. We would then have been embarrassed in the face of expressions which look like individual names, but which are either vacuous or "denote too much". How should they be classified? To say that individual names may be vacuous would be self-contradictory.

Positive and negative names.⁷

We shall first attempt to give a definition of negative names. The English language contains words and prefixes which are used in negations, for example, "no", "not", "in-", "im-", and "un-". Let us call such expressions "*signs of negation*". We shall now define positive and negative names.

Definition of positive names, D. P. N.:

t is a positive name =_{def} t is not a negative name.

Definition of negative names, D. N. N.:

t is a negative name =_{def} t is an expression which contains an odd number of occurrences of signs of negation.

a) The clause "odd number of occurrences of signs of negation" in D. N. N. is added to exclude that expressions such as "no immortal" be classified as negative names. Such a classification would be queer, because such expressions are contextually synonymous to expressions which do not contain any sign of negation at all, and which ought to be classified as positive. "No immortal", for example, is synonymous with "mortal". It is undesirable that synonymous expressions should be classified in different classes.

b) "Odd number of occurrences of signs of negation" means in praxis "just one occurrence", for it is most improbable that anyone should use as many as three signs of negation in an expression before a copula or after a copula, i.e., in a name.

c) Negative names of the form "no ——" are first names. Names of the form "not ——" are second names.

⁷ E. W., I. 2. 7-8. — Thomas Hobbes, *Leviathan or the matter, form and power of a common-wealth, ecclesiastical and civil*. The text ed. by A. Waller, Cambridge 1904. (Cambridge English Classics.), Ch. 4, P. 1.

Semantic properties of negative names. Hobbes declares that a negative name denotes every object which is not denoted by the corresponding positive name. This account is not satisfactory for negative names which appear before the copula. Consider the sentence:

"No man is 200 years old." (S₁₂)

If the first name denotes every object which is not a man, then S₁₂ ought to be synonymous with:

"Every object which is not a man is 200 years old." (S₁₃)

S₁₂ and S₁₃ are obviously not synonymous.

Hobbes has no other account of negative names before a copula than this.

Hobbes thinks that it is possible to transform sentences in which the first name is negative into logically equivalent sentences in which the first name is positive. To show this I shall make use of modern logic. In fact it holds:

No A is B = $(\bar{\exists}x)(Ax \cdot Bx) = (x)(\bar{A}x \vee \bar{B}x) = (x)(Ax \supset \bar{B}x) =$
= All A are not B.

The sign of equality means here "is logically equivalent to."

Hobbes has no means to prove this equivalence. He most certainly cannot do it within his theory of truth; for this theory cannot be applied when the first name is negative, as we have just shown.

Hobbes account of the denotational properties of second names which are negative, is more plausible. For example, the sentence:

"Socrates is not Italian." (S₁₄)

should according to Hobbes' account have the same meaning as the sentence:

"Socrates is one of those objects which are not Italian." (S₁₅)

II. *Theory of Truth*

In E. W. I. 3. 1., Hobbes states that propositions are sentences used to affirm or to deny, and that they "express truth and

falsity". In other words, propositions are the same as cognitive sentences. In the next section⁸ there is a passage labelled "Propositions defined". "A proposition is a speech consisting of two names copulated, by which he that speaketh signifies he conceives the latter name to be the name of the same thing whereof the former is the name; or (which is all one) that the former name is comprehended by the latter."

Not all cognitive sentences have the form $a \text{ e } \beta$, where a and β are names and e is a copula. The definition is therefore too narrow if it is regarded as a lexical definition. Hobbes here selects for study a special set of cognitive sentences, viz., those sentences which have the form $a \text{ e } \beta$. Those sentences he calls "propositions".

This is unfortunate because, as we saw, the term "proposition" is also used in a wider sense by Hobbes.

How are other cognitive sentences related to the special set selected by Hobbes? He writes:⁹ "... every proposition may be and uses to be, pronounced and written in many forms." "Different forms of a proposition are said to be equipollent".¹⁰ Hobbes holds that at least some cognitive sentences which are not of "standard form" are equipollent (=logically equivalent) to sentences of the form $a \text{ e } \beta$. Or perhaps all? It is not possible to make out how Hobbes would answer this question. He seems to hold that his definition of truth, which explicitly refers to his standard sentences only, nevertheless is relevant for every cognitive sentence. On the other hand Hobbes has nowhere explicitly committed himself to the risky thesis that *every* cognitive sentence is logically equivalent (=equipollent) to some sentence of standard form. It is therefore preferable to judge that Hobbes is open-minded about the scope of this theory of truth.

We shall now attend to Hobbes' concept of equipollence. No definition of this relation is stated, but Hobbes gives many examples of couples of sentences which, according to him, stand in the relation of equipollence to each other. Those sentences are

⁸ E. W., I. 3. 2.

⁹ E. W., I. 3. 12.

¹⁰ *Ib.*

also logically equivalent to each other. In some cases we find Hobbes giving reasons for judging two sentences to be equipollent.¹¹ In all those cases both sentences are of standard form, and Hobbes makes use of his definition of truth. In those cases, however, where one of the sentences is not of standard form Hobbes gives no support for his claim that the sentences are equipollent. We must therefore conclude that Hobbes does not possess the proper means for extending his theory of truth beyond the confines of sentences of the form $\alpha \text{ e } \beta$. In order to make such an extension he would need to have a theory of logical equivalence which is *independent* of his theory of truth as applied to standard sentences.

An extended theory of truth would contain the parts roughly indicated below:

Outline of definition of equipollence:

p is equipollent to q if and only if - - -

Outline of definition of truth for standard sentences:

If p is a standard sentence then p is true if and only if
- - -

Definition of truth for sentences which are not of standard form:

A sentence p which is not of standard form is true if (but not only if?) there exists a true sentence of standard form q, which is equipollent to p.

A question of importance for such a wider theory of truth is this. Does there exist sentences which are not logically equivalent to any standard sentence?

As matters stand, Hobbes has only given us a theory of truth for his standard sentences. His discussions of other sentences are loose appendices to that theory.

The central passage in Hobbes' theory of truth is the following one¹² (already quoted twice). "A proposition is a speech consisting of two names copulated, by which he that speaketh signifies he conceives the latter name to be the name of the same

¹¹ E. W., I. 3. 14-16.

¹² E. W., I. 3. 2.

thing whereof the former is the name; or (which is all one) that the former name is comprehended by the latter."

I shall make Hobbes' statement somewhat more precise. A sentence of the form $\alpha \in \beta$ is true, so Hobbes says, if whatever is denoted by α is also denoted by β . β may or may not also denote objects which are not denoted by α . If β denotes whatever α denotes then β is said to *comprehend* α . This theory may be called *Hobbes' comprehension theory*.

The comprehension theory of truth is essentially a definition of truth and discussions clustering around that definition. Definiendum for the definition of interest is "true" in the context "A sentence of the form $\alpha \in \beta$ is true . . ." Definiens is: The name " α " is comprehended by the name " β ". The definition of truth is thus:

Definition of truth for sentences of the form $\alpha \in \beta$, D. T.:

A sentence of the form $\alpha \in \beta$ is true =_{def.} The name " α " is comprehended by the name " β ".

The definition may alternatively be regarded as a schema of innumerable definitions obtained by replacing α and β by name-tokens. It is plausible to require for the adequacy of the definition of truth that the results of such operations be significant and reasonable. Is this the case?

a) We have seen that Hobbes has not accounted for the denotational properties of name-tokens of the form "no —". Consequently, we do not know how to make sense of definientia such as "The name "no man" is comprehended by the name "immortal"."

b) Vacuous name-tokens present also difficulties. Is a vacuous name comprehended by any name? No matter how we answer this question we will run into difficulties. Suppose we answer the question in the affirmative. Then there would be no way of distinguishing between cases where a name comprehends and cases where a name does not comprehend a vacuous name. Every name would comprehend a vacuous name. In particular both α and not β would comprehend a vacuous name α . Hence the following two sentences would be true, provided α is vacuous.

" a is β " and " a is not β ."

But such sentences ought to have different truth-values.

On the other hand a negative answer to the question would lead to the consequence that all sentences with a vacuous first name would be false. Hence, in particular, " a is β " and " a is not β " would both be false in contradiction to an established logical principle, which Hobbes accepts.

A third alternative would be to declare that vacuous names are neither comprehended nor not comprehended by other names, but this very answer is in conflict with the principle that contradictory sentences have different truth values.

We may look upon the difficulties concerning vacuous names also from another angle. Hobbes' comprehension theory entails that the formula (a is β) or (a is not β) depends for its truth, and that the formula (a is β) and (a is not β) depends for its falsity on the *factual* question whether a does or does not denote.

We have now argued that D. T. has a low degree of adequacy according to the criterion that every instance of the definition schema obtained by replacing the variables a and β by name-tokens in the formula D. T. ought to be significant and reasonable.

It is evident that the scope of the comprehension theory of truth, although it is already rather narrow, must be restricted still further; and that the theory must be complemented not *only* with regard to sentences *outside* the set of standard sentences.

We shall now discuss the question: What is the function of the comprehension theory of truth?

Given a *special* sentence p , we may ask these questions:

- (1) What does p state?
- (2) What do we mean when we say that p is true?
- (3) What condition must obtain in order for p to be true?
- (4) How do we find out whether p is true or not?

If p is the sentence:

"Socrates is a Greek."

(S₁₅)

I think that it is good common sense to answer these questions in this way:

- (1)_{c.s.} *that* Socrates is a Greek.
- (2)_{c.s.} *that* Socrates is a Greek.
- (3)_{c.s.} Socrates *is* a Greek (or: It is the case that Socrates is a Greek.)
- (4)_{c.s.} We may inspect an encyclopedia or an authoritative history of philosophy.

Notice that (1) and (2) are answered in the same way: The answer is a *that*-clause. (3) is answered by producing a sentence token of the same type as *p* in an assertive tone of voice.

None of my common sense answers to the questions stated is like Hobbes' definition of truth. What answers would Hobbes give? I believe he would say:

- (1)_H *that* "Socrates" is comprehended by "a Greek".
- (2)_H *that* "Socrates" is comprehended by "a Greek".
- (3)_H "Socrates" is comprehended by "a Greek".
- (4)_H We examine whether "Socrates" is or is not comprehended by "a Greek".

The answer (3)_H is an instance of the definiens in D. T.

I am therefore inclined to conclude that the various instances of Hobbes' schema D. T. are answers to questions like (3), but not to questions like (1), (2), or (4). Furthermore, I think that Hobbes' answer to (3) is rather different from a common sense answer ((3)_H ≠ (3)_{c.s.}).

Hobbes' comprehension theory of truth invites us to make the following reflections.

A sentence is analytic, if it is true in virtue of its meaning. Hobbes' theory implies that true sentences of the form $\alpha \in \beta$ are true in virtue of the denotational properties of their names. Hobbes, as a consequence, committed to hold that all true sentences within the scope of his theory of truth are analytic? Hobbes himself distinguishes between necessary and contingent sentences, a distinction which has close affinity to that into analytic and syntetic ones. He says:¹³ "A necessary *proposition* is where nothing can at any time be conceived or feigned, whereof the

¹³ E. W., I. 3. 10.

subject is the name, but the predicate also is the name of the same thing . . ." - "... a *contingent* proposition is that, which at one time may be true, at another time false . . .»

These definitions are not very clear to be sure. However it is obvious that Hobbes would feel unhappy if his theory of truth would commit him to classify all true standard sentences as analytic ones. Is Hobbes committed to such a contention? I think not. Hobbes distinguishes between *denotation* and *connotation* (or signification) of names. He says that names denote objects and connote (or signify) concepts.¹⁴

The difference between necessary and contingent propositions is a difference between the ways in which concepts are related to each other, not a difference between the denotations of the names appearing in them.

Hobbes distinguishes between complex and simple concepts. I think that the following definitions of analytic and synthetic concepts fit well into Hobbes semantics:

Definition of analytic sentences of standard form, D. A. S.:

A true sentence of the form $\alpha \text{ e } \beta$ is *analytic* if and only if the concept connoted by α is identical with, or contains as a part, the concept connoted by β .

Definition of synthetic sentences of standard form, D. S. S.:

A sentence of the form $\alpha \text{ e } \beta$ is *synthetic* if and only if it is not analytic.

An example of an analytic sentence is this:

"Man is a living body." (S₁₆)

An example of a synthetic sentence is this:

"Ravens are black." (S₁₇)

"Man" in S₁₆ connotes a concept which consists of the concepts connoted by the names "living", and "body".

On the other hand "Ravens" in S₁₇ connotes a concept which does not contain as a part the concept connoted by "black".

¹⁴ E. W., I. 2. 5.

It is not my task in this paper to enter any further into Hobbes' theory of concepts and of connotation. Hobbes holds that concepts are mental occurrences (ideas).¹⁵ His theory of concepts has a close affinity to those of the classical British empiricists.

¹⁵ Thomas Hobbes, *The elements of law, natural and politic*. Ed. by F. Tönnis. London 1889, p. 26. "For the truth of a proposition is never evident until we conceive the *meaning* of the words or terms, whereof it consisteth which are always *conceptions of the mind*." (My italics.)

The logical status of meta-ethical theories

by

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My aim here will be to discuss the logic of meta-ethical theories which attempt to analyse ethical sentences in terms of other types of sentences or other types of human activity. That is, I shall discuss the logical status of statements like: "Value judgements are simply expressions of emotion", "Value judgements are (disguised) commands", "Ethical statements are (peculiar) descriptions", "Ethical statements are scientific statements of fact", etc. I shall first consider the statement: "Value judgements are simply expressions of emotion"¹ as this position, the Emotive Analysis, has proved the most influential of all such theories.

The statement: "Value judgements are simply expressions of emotion" is open to three interpretations. It could mean (a) that the class of value judgements is identical with the class of expressions of emotion, or (b) that the class of value judgements is properly included within the class of expressions of emotion, or (c) that a newly defined term "expression of emotion" is being introduced and that the class of "expressions of emotion" includes the classes of expressions of emotion and value judgements.

It will be seen that if (b) is false, (a) is also false and as it is the literal falseness of the theory in which I shall at first be in-

¹ This is part of a longer quotation from Ayer which reads as follows: "In so far as statements of value are significant, they are ordinary 'scientific' statements; and in so far as they are not scientific, they are not in the literal sense significant, but are simply expressions of emotion which can be neither true nor false." Ayer, A. J., *Language, Truth and Logic* (1935), New York, Dover, 1946, Ch. IV, p. 102.

terested, it is sufficient for me to consider (b). Interpretation (c) is a horse of an entirely different colour and will be considered later.

My stand on interpretation (b) is not only that it is false, but that it is *obviously* false. I strongly suspect it became such a very influential theory on the philosophically dubious grounds of introspection. In the confused realm of introspection it is undoubtedly difficult to decide whether, having given vent to a complex sign of an ethico-emotive nature, we are expressing a value judgement in the sensible form of an expression of emotion, or whether we have expressed an expression of emotion in the sensible form of a value judgement, or whether we are suffering from a liver condition. This, of course, only occurs when we are strongly emotionally stirred, but it is an easy step to generalize from here to the more sober occasions when we express a value judgement without emotional disturbance. But because we find it hard or impossible to decide in individual cases quite what we *expressed* by any given sensible sign, it is surely not as difficult to detect differences in the logic of given sensible signs. Nor is it illegitimate or question begging to consider the logic of the sensible sign with no particular regard for what any particular individual was expressing when he gave vent to it. "You left the door open" is a simple factual statement and should be analysed as such even though, in the Arctic perhaps, much might be expressed in making the statement.

If I am correct in supposing that introspection, and a philosophically irrelevant use of introspection, is the source of belief in the Emotive Analysis, then it should be easy to show that however much value judgements and expressions of emotion may have properties in common, the former is not included within the latter. And it is, I think, easy. For the negative of a value judgement makes sense where the negative of an expression of emotion does not. Gurgles, screams, blushes, exclamations like "Damn!" – these do not have negatives. But "It is wrong to steal" – "It is not wrong to steal" – these make perfectly good sense. We cannot argue with a gurgle or a "Damn!" though we may disavow them and we may argue with the advisability of even

giving vent to them. We can and do argue with: "This is a bad book". It is not my concern here whether the argument is merely leading to agreement as to belief about the facts, whether the possibility of logically sound ethical argument exists, or any such abstruse matters. Argument is possible and occurs, physical persuasion is possible, because the form of words "I was wrong and you were right" is applicable. None of which is the case with expressions of emotion. We can say: "I shouldn't have, was ill advised to have, said 'Damn!'". We cannot say: "All right, 'undamn'", the way we can say: "All right, it is not wrong to steal".

Such *legalistic* considerations as these cause difficulties. For though I do not see how anyone could deny their cogency, they tend not to strike at the root of belief in the Emotive Analysis. This, as I have suggested, comes possibly from the non-logical basis for belief in the Emotive Analysis.² Caught in the squeeze between interpretation (c) which, depending as it does on redefinition of a key term, could easily be trivial and uninteresting even if true,³ and the impossibility of holding to (b), supporters of the Analysis are naturally dissatisfied. It might be claimed that I have taken the Emotive Analysis too literally, that I have been unsubtle in my approach, boorish, in fact. Perhaps. But if value judgements are not included within the class of expressions of emotion when the statement is taken to mean what naive (ie unphilosophic, ie without artificial redefinition) inspection would indicate, then interpretations (a) and (b) are false. And once it is only interpretation (c) which is being offered by it, the statement: "Value judgements are simply expressions of emotion" becomes much less startling and exciting than it first sounded. The statement: "A candle flame lives", made by a

² For those who are unconvinced or interested, I have put forward my objections to the Emotive Analysis in more detail in 'A Note on the Emotive Theory' to appear in a forthcoming issue of *Philosophy*.

³ Just because a position depends on the redefinition of a key word, it does not follow that it loses all philosophic merit. What value it has then depends on how "expression of emotion" is redefined, for this could be done to make the Analysis very interesting and enlightening or entirely trivial. Cf p. 80 f. of this article.

sane scientist, becomes far less startling and exciting when we are told that, to be true, the word "lives" in the statement must be redefined to mean "continues in a state of disequilibrium".

There is more to the situation as regards the Emotive Analysis than this. The enormous influence which the Emotive Analysis has wielded is due, also, we may surmise, to the fact that not very great violence has to be done to the conventional meaning of "expression of emotion" to make the Emotive Analysis true. Proponents of the theory might wish to say that the meaning of "expression of emotion" has only to be extended slightly to make their analysis true, that redefinition is unnecessary. They might even add that the words "expression of emotion" are themselves semi-technical and have very little meaning outside a philosophic context. All this is true. However, it does not alter the fact that the Emotive Analysis is startling and exciting precisely because, hearing the term "expression of emotion" for the first time, we would emphatically not expect it to cover value judgements, and, if my contention is correct, rightly not expect it to cover value judgements. Thus in so far as the position is startling and exciting (for this tends to stop once redefinition of any kind takes place), it is false.

In an unguarded moment we might tend to feel that the truth about value utterances lies *between* the very heavy demands on the class of expressions of emotion including the class of value judgements and the almost (or, perhaps) trivial situation where some newly invented class "expressions of emotion" is so defined that it includes the class of expressions of emotion and the class of value judgements. Interpretation (b) is too strong, admittedly, but interpretation (c) is far too weak.

It is when we are faced with this difficulty that our inquiries become so badly entangled. On the one hand we have the terms "expression of emotion" and "value judgement" and on the other screams, gurgles, blushes, exclamations like "Damn!" etc. and statements like "X is good", "Y is the highest of its kind", etc. If we let "expressions of emotion" mean just and only screams, gurgles, blushes and exclamations like "Damn!", and "value judgements" mean just and only statements like "X is good", "Y

is the highest of its kind", etc., we would have gained nothing but a form of shorthand. Further, a misleading shorthand for it is so very difficult to keep in mind all these terms must cover. We gain nothing for if screams, gurgles, blushes, exclamations like "Damn!", etc. are not the same as sentences like "X is good", "Y is the highest of its kind", etc., then, equally it makes no difference what gyrations we perform, our shorthand terms cannot logically be the same, ie the proposition: "Value judgements are simply expressions of emotion" cannot be true so long as these definitions stand.

On the other hand, we may define our philosophic terms so that they refer to abstractions from screams, gurgles, blushes, exclamations like "Damn!", etc. and from "X is good", "Y is the highest of its kind", etc. That is, of (say) screams, gurgles, blushes, exclamations like "Damn!", etc. all have the properties a, b, c, d, e, & f, we might define "expression of emotion" as anything having properties a, b, c, & d. And similarly for "X is good", "Y is the highest of its kind", etc. and "value judgements". Then our newly defined abstractions "Value judgement" and "expression of emotion" might be identical or one might include the other. But from this it would not follow that "X is good", "Y is the highest of its kind" etc. are the same as or equivalent to screams, gurgles, blushes, exclamations like "Damn!" etc., ie the proposition: "Value judgements are simply expressions of emotion" would, in isolation, still not be true – it would only be true if it read: "Value judgements are simply expressions of emotion when 'value judgement' is defined thus and 'expression of emotion' is defined thus". That is, we are left with the situation which naive inspection should have given us in the first place: that the proposition: "Value judgements are simply expressions of emotion" is false unless "value judgement" and "expression of emotion" are so defined as to make it true – which is the case with any false proposition whatever ie is true of the proposition: "Men are simply fish with legs" or the proposition: "Atoms are simply very small golf balls."

We are left, then, with this dichotomy. Either it is true to say that value judgements are simply expressions of emotion with

the proviso that "value judgement" and "expression of emotion" are defined in such a way that these terms denote *something like*, or denote an abstraction *having properties in common with*, sentences like "X is good", "Y is the highest of its kind", etc. and screams, gurgles, blushes, exclamations like "Damn!" etc. respectively. Or it is *not* true to say that value judgements are simply expressions of emotion where these terms stand for sentences like "X is good", "Y is the highest of its kind", etc. and screams, gurgles, blushes and exclamations like "Damn!" respectively, but it is true to say that value judgements are something like expressions of emotion or value judgements have properties in common with expressions of emotion. (But we must remember that a Coral snake is something like a twig and that a whale has properties in common with a cat.) Nor is there any way out of this dichotomy.

I shall now put the same point in symbolism both to facilitate further discussion and clear up point of ambiguity in the inevitably complex discussion just presented.

I shall use the symbol ' \subseteq ' to denote inclusion, ' \supseteq ' to denote "is something like" or "has properties in common with", and ' \longleftrightarrow ' to denote identity. I shall use the symbol 'V' to denote sentences like "X is good", "Y is the highest of its kind", etc. and 'Vd' to denote anything covered by the definition of the term "value judgement" however it might be defined. Similarly, I shall use 'E' to denote screams, gurgles, blushes, exclamations like "Damn!" etc. and 'Ed' to denote everything covered by the definition of the term "expression of emotion" however it might be defined.

Then, from our previous discussion, we can see

$$V \subseteq E \quad \text{is false} \quad (\text{i})$$

but that

$$V \supseteq E \quad \text{is true} \quad (\text{ii})$$

Then, if we make definitions so that

$$E \supseteq Ed \quad (\text{iii})$$

and $V \rightleftharpoons Vd$ (iv)

we may easily be able to prove that

$$Vd \subseteq Ed \quad (v)$$

But this gets us nowhere. From it we could, at most conclude that $V \rightleftharpoons E$ (ii)

which is what we started from.

Alternatively if, by a process of definition, we make

$$E \longleftrightarrow Ed \quad (vi)$$

and $V \longleftrightarrow Vd$ (vii)

we know it is useless to even try to show that:

$$Vd \subseteq Ed \quad (viii)$$

as we already know that

$$V \subseteq E \quad \text{is false} \quad (i)$$

from which, and (vi) and (vii), it follows that

$$Vd \subseteq Ed \quad \text{is false} \quad (ix)$$

There are more variations than those given there (ie, in the first example, (iv) could have read

$$V \subseteq Vd$$

without altering the conclusion), but it is clear, I think, that all would give the same uninteresting results.

Before we go on to discuss the more general conclusions concerning all meta-ethical theories which attempt to analyse ethical sentences into other types of sentences or other types of human activity, there are some things which may be said about the symbol ' \subseteq '. Which is to remark, as a general empirical proposition, that few if any philosophical propositions which make any stir are propositions of this type – for such a proposition is usually obvious if it concerns matters of language with which we are at

all familiar. And philosophers usually reserve their pronouncements about current language for propositions which do not seem to be obvious. If we are completely familiar with the characters and activities of Mr X, who goes sometimes by the alias "Mr Y", we are in no doubt that the terms "Mr X" and "Mr Y" denote the same man. The only point of using an alias is when many people are not familiar with all or most of our activities *which is never the case as regards language in everyday use*. The whole point of our discussions about everyday language is that we are all excellently versed in it. As Wisdom says in 'Philosophy, Anxiety and Novelty': "Every philosophical question is really a request for a description of a class of animals – of a *very* familiar class of animals. That is my point, that the classes of animals are very familiar to us all. Consequently philosophical answers are descriptions of very familiar classes of animals."⁴

I now wish to show that my argument as regards the Emotive Analysis applies to all propositions like: "Value judgements are (disguised) commands", "Value judgements are descriptions" (e.g. "goodness" is a pure, unanalysable quality like yellowness) "Value judgements are scientific statements of fact", etc. and of course, other types of analyses than meta-ethical analyses though the ethical analyses are all I am interested in here. If the symbol for descriptive sentences is 'D', for commands is 'C', for scientific statements of fact is 'S', then it is clear that:

$$V \subseteq D \quad (\text{x})$$

$$V \subseteq C \quad (\text{xi})$$

and

$$V \subseteq S \quad (\text{xii})$$

are all false. Equally

$$V \supseteq D \quad (\text{xiii})$$

$$V \supseteq C \quad (\text{xiv})$$

⁴ Wisdom, John, 'Philosophy, Anxiety and Novelty', *Philosophy and Psycho-Analysis*, Oxford, Blackwell, 1953, p. 112. Reprinted from *Mind*, Vol. LIII, No. 210, 1944.

and $V \supset S$ (xv)

are all true – just as we saw that

$V \subseteq E$ is false (i)

and $V \supset E$ is true (ii)

for 'V', 'D', 'C', 'S' have *something* in common, however rudimentary this may be.

Further, as an empirical observation, it seems most unlikely, especially in view of the other empirical observation discussed in the previous paragraph, that any proposition of the form:

$V \subseteq P$ (xvi)

where 'P' is a type of sentence (ie a description, command, question, etc.) or some other form of human activity (ie an expression of emotion, exclamation etc.) is ever likely to be true as we would surely have noticed it by now if there were identical types of sentences going under two separate names. To look at it another way, would hardly have bothered to call identical types of sentences by different names.

From which, I suppose, we could prosaically conclude that "every type of statement has its own type of logic". But there is more meat on this bone than that. There are two divergent lines of thought which could spring from the situation as I have presented it here. Let us imagine a metaphysician who gives his support to the Emotive Analysis. At this juncture, he could say: "You have but mistook me all this while. When I say, 'Value judgements are simply expressions of emotion', I do not mean that any given value judgement can be analysed into expressions of emotion. I mean this no more than I meant we cannot arrange to meet outside the post office at six when I said that time does not exist. What I mean is that attempts to analyse value judgements in the past have almost always been in analogy to some sort of description, command or what have you, while I am maintaining that a better analogy is to our emotive responses to situations. Since you have tied me down in this unpleasant manner, saying either that 'expression of emotion' shall be redefined,

making my position verge on the trivial, or that it shall not be redefined, making my position false, I will admit that I never took myself seriously when I tried to 'prove' that value judgements were simply expressions of emotion.⁵ The aim of my 'proof' was to point up the very powerful analogy between our emotional response to a given situation and the sort of value judgement we are apt to pass on it. To emphasize this analogy had a double point. In the first place we had had enough of the other analogies and the Emotive Analysis was a refreshing change. In the second place, it was not that philosophers who supported other analogies had *chosen* to ignore the emotive response features of our value judgements; they seemed to be genuinely *blind* to their existence. To cure them of their blindness, to make them see what before they had failed to see, I posed my own analogy. The new view I proposed was not only interesting, it was essential as a therapeutic to cure a far too one sided view. I had no desire that others should flock to my standard and agree with me, I wanted them to *take account* of the point of view I was putting forward. And I am very pleased with the results I have achieved. I wrote nonsense, but wasn't it revealing nonsense? (We may notice that a very similar argument could be put forward by a metaphysician who supported any of the other analyses I have been discussing in this article.)

This, put very much less frankly, is, I suspect, what Ayer is saying in 'On the Analysis of Moral Judgements'.⁶ Nor would I criticise this point of view, be it Ayer's or not. I would go on to discuss it and note its value at some length had this not been done so admirably by Wisdom in almost everything he has published in the last ten years (cf. almost all the articles in *Philosophy and Psycho-Analysis*).

We have seen, then, that positions like: Value judgements are commands, expressions of emotion, descriptions, etc. have been revealing because they have pointed up similarities and properties

⁵ Or, "I was so carried away with the idea that I did not notice that a rigorous proof was impossible".

⁶ Ayer, A. J. 'On the Analysis of Moral Judgements', *Horizon*, vol. xx No. 117, 1949. Reprinted in *Philosophical Essays*, p. 241.

which we had not before noticed. But this need not be the last word, not by any means. There is another line of thought open to us.

At the end of the analysis of one of these propositions we are left with a proposition of the form

$$V \Rightarrow P \quad (\text{xvii})$$

which is true. But it could be maintained that this is entirely *insufficient* as a final philosophic position for the symbol \Rightarrow is is so *very* vague. Granted that arguments brought in support of propositions of the form

$$V \subseteq P \quad (\text{xvi})$$

though they do not succeed in proving it, give us a far more precise insight into the logic of value judgements than we had previously, but even this is probably insufficient. Probably insufficient because a few crucial arguments or statements are often considered sufficient to prove the point desired (cf. Ayer's original argument for the Emotive Analysis in *Language, Truth and Logic*) where to discover the exact similarities a far longer, more exhaustive approach is required. One way in which to do this is to give all the properties the logic of the two types of statement in question hold in common a special name (eg interpretation (c) of our original analysis of the Emotive Analysis) when the process of defining the special name will list at least some of the logical properties of value judgements. This would provide a partial answer to the demand for an analysis of value judgements. But it would not be a full answer for it would not preclude the possibility of finding (other) logical properties which value judgements hold in common with other types of statement and giving them a special name. These two positions, were they put forward, would be non-contradictory if the philosophers making the analysis knew their business and made no logical slips. If we set down the properties which value judgements hold in common with every other type of sentence, we would have a fairly complete analysis. But not a complete one. For it is possible, I would say likely, that value judgements have some properties which

they hold in common with no other types of statement and under this type of analysis these properties would remain forever hidden.

However, there is another approach, psychologically speaking, a very difficult approach for the philosopher concerned. Once we have reached the stage of naming the properties which value judgements have in common with other types of statement, the whole business of finding properties which value judgements have in common with other types of statement becomes a useless trapping. As a psychologically necessary "starter" or a logical pump "primer" it may be desirable, but it holds no necessary place in a discussion of the logic of value judgements. In other words, in so far as we wish to set down the logic of value judgements, we should set out on a process of description, describing and detailing the properties which value judgements possess. After this process is complete we might wish, as an educational aid or as part of the process of building a language map, to put the matter more graphically by pointing out that value judgements hold these properties in common with commands, these with expressions of emotion, these with descriptions, and so on; and perhaps that, say, commands, value judgements and expressions of emotion all have this and that property in common.

Philosophers *might* do this. I am not myself sure that it would be a good idea for them to do so. However, if they did, we would achieve one thing: we would have turned at least part of ethical investigation into the detailing of the logical geography of our language and thus justify those philosophers who have been saying for so long that this is just and only what it is.

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Ifs and Cans¹

by

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If someone were to ask what idea is expressed by the verb 'can' in all its moods and tenses, he might be told that it expressed power, potency or potentiality, capacity, capability or ability, contingency or possibility – all or some of these. But this answer, though true enough, is unenlightening, and unenlightening just because it is so obviously true. If there are puzzles about 'can' there will be puzzles about these. Human actions, the things people actually do, are, we feel, ground-floor members of the world; their abilities are not. Philosophers have often felt that particular statements, expressed by sentences in the indicative mood with names of entities as subjects, are somehow paramount. We use the indicative mood to say flatly that something is or was or will be the case. Hypothetical and universal statements, by contrast, are suspect. This feeling has led some philosophers (for example some logicians who have been incautious about the relation between the logical constants of a language and those of a calculus) sadly astray. It is certainly wrong to say flatly that 'all mules are sterile' means the same as 'if anything is a mule, it is sterile' or 'nothing is both a mule and non-sterile'; but the idea that it is not sufficiently categorical to get in on the ground floor but requires to show its connections with true-blue categoricals about this and that mule before it can be admitted is not wholly erroneous. Likewise statements about the next mule are not wholly irreproachable, and the drive to analyse universal statements into hypotheticals and then treat these hypotheticals as

¹ A reply to J. L. Austin: *Ifs and Cans*. British Academy Lecture, Oxford University Press, 1956.

truth-functional sets of particular categoricals is not wholly to be resisted.

There is a notorious connection between universal statements and statements made with 'can' which leads us to suspect that the latter, though in one sense categorical enough, are not wholly flatly and irreducibly categorical. One of the many features which these two types of statement have in common is that, while they must always be backed in the end by statements as to what is or was the case, they always assert more than this. We can see Tom and Dick playing bridge, but we cannot see the class of all bridge players, and in much the same way, though we can see someone playing bridge and perhaps see him manifesting his ability to play bridge, we cannot see his ability. Abilities are somehow parasitic on performances, if only because an ability to do something is an ability to *do* that thing.

This is one reason why philosophers have so often tried to analyse can-statements in terms of if-statements, a type of analysis which it is the main purpose of Austin's paper to defeat. Their attempts have often been complicated and confused by two factors. (i) They have, as Austin plainly shows, confused analysis with supplementation, the idea that 'X can do Y' must be *analysed* into some statement in which the word 'can' does not occur (for example 'X will do Y, if he tries'), with the idea that 'X can do Y' is always *incomplete* as it stands, that it always means something of the form 'X can do Y, if . . .', a form in which the word 'can' is still present. (ii) The topic is usually raised in the context of free will and responsibility; and here it is usually 'could have' that requires analysis. On the face of it, it is much more plausible to maintain that 'could have' requires analysis in terms of or supplementation by an if-clause than that the whole verb 'can' in all its moods and tenses does so. Hence philosophers (including myself) have confused considerations which might lead one to give an hypothetical analysis of 'could have' with those that make for an hypothetical analysis of 'can'.

On the first point, both in *Ethics* and in this paper I am concerned to argue that 'can' requires analysis in terms of 'if', not that it requires supplementation. Moore was quite wrong when

he said that 'I could have walked a mile in 20 minutes' means 'I *could* have walked a mile in 20 minutes, if I had chosen'. To avoid the second error I shall start here with 'can' and move to 'could have' later.

There are two points in Austin's paper which may be accepted from the start. (i) There are at least three senses of 'can', which Austin calls the 'ability', the 'opportunity' and the 'all-in' senses. (ii) The form 'could have' is often, not a subjunctive, but the past indicative of 'can' in any of these three senses. It means, not 'would have been able, if ...' but categorically '*was* able'. In the first part of this paper I shall discuss the relations between the three senses of 'can'. Austin's view here is that there is "an all-in, paradigm use, around which cluster and from which diverge, little by little and along different paths, a whole series of other uses",² among them 'ability' and 'opportunity'. In the second part of the paper I shall argue that, in all three senses, the whole verb 'can' requires to be analysed in terms of 'does ... , if ...'.

I

My first thesis, then, is that the all-in 'can' (to be *fully* able) is a conjunction of ability and opportunity. These are umbrella-like words covering a host of things such as 'having the strength, skill, know-how etc.' and 'being in a position to, having the means or equipment to etc.' So my first thesis is still highly schematic in character. A full treatment would have to go into these differences and I suspect that it might emerge that ability and opportunity are not as clearly distinct *genera* as we usually take them to be.

That the all-in 'can' is a conjunction of ability and opportunity emerges, I think, from what Austin himself says about the case in which 'he could have done X' really is a conditional, requiring, to give its full sense explicitly, to be completed by an if-clause, typically 'if he had had the ability' or 'if he had had the opportunity'.³ 'He could have smashed that lob, if he had been any good

² *op. cit.* p. 130.

³ *op. cit.* p. 129.

at the smash' is a conditional; yet, according to Austin, "it nevertheless manages to assert, by means of its main clause, something categorical enough, that he *did* have a certain opportunity". We must ask here which of the three senses of 'can' is being used in the main clause. It cannot be the 'can' of ability; for if it were the whole sentence would read 'He would have had the ability, if he had had the ability', and it is clearly not this tautology that we mean to assert. Nor can it be the 'can' of opportunity; for in that case the whole sentence would read 'he would have had the opportunity, if he had had the ability'. But according to Austin we assert categorically that he *did* have the opportunity, not that he *would* have had it if he had had the ability; the opportunity was there whether or not he had the ability to make use of it. It must, then, if the statement is genuinely conditional, be the past subjunctive of the all-in 'can' that we are using here. For clarity we may re-write the sentence as 'he would have been fully able, if he had had the ability'. Now this certainly does imply, even though it does not state, something that is "categorical enough". For it implies that he actually had everything other than ability which is necessary for being fully able, that he had, among things, an opportunity to smash that lob.

These categorical implications of 'he could have done X, if he had had the ability' and of 'he could have done X if he had had the opportunity' lead Austin to suggest that they are not conditional sentences at all. He has already introduced us to an 'unorthodox' type of if-sentence, exemplified by 'there are biscuits on the sideboard if you want them', which does not express a conditional and for which the ordinary logical rules for conditionals do not hold.⁴ Perhaps, then, these sentences *just* assert respectively opportunity and ability, their if-clauses being of the unorthodox type. To reinforce this suggestion Austin tells us that 'I

⁴ *op. cit.* pp. 113-5. The main features of these unorthodox cases are that they do not entail their contrapositives but do entail their own main clause. From 'there are biscuits on the sideboard if you want them', we cannot infer 'if there are no biscuits, you do not want them', but we can infer 'there are biscuits on the sideboard'.

can' in the ability sense may be expressed by 'he can in the full sense if he has the opportunity' and that when 'he can' is used in this way no one would take it to be a subjunctive or conditional. If therefore, the 'could have' in 'he could have smashed that lob, if he had had the ability' is the past indicative tense of 'can', that statement also will not be a conditional. This is true, but irrelevant. For it is not clear that 'could have' in the two hypothetical-seeming examples under discussion *must* be taken to be the past indicative of a restricted sense of 'can'. Why should it not be, what it certainly seems to be, the past subjunctive of the all-in 'can'? And, if this is what it is the statements will be genuine subjunctive conditionals equivalent to 'he would have been fully able, if he had had the ability' and 'he would have been fully able, if he had had the opportunity'.

The question, then, is this: when we use these sentences are we *just* making categorical assertions of opportunity or ability – the if-clauses adding something indeed, but not imposing a condition – or are we making an hypothetical assertion, from which a categorical can indeed be deduced, but which cannot be identified with that categorical? It may be that these two come to the same thing for all practical puposes, just as ' q ' and ' $p \vee q$ ': ' $\neg p$ ' come to the same thing; but it is important to see whether or not they do. Suppose that there were another restricted sense of 'can' which we will call ' t ' and that it seems likely that 'he can (all-in)' means 'he has the ability and the opportunity and t '. There will now be a great difference between 'he would have been fully able, if he had had the opportunity' and the simple categorical 'he had the ability'. The categorical will still follow from the hypothetical; but the converse inference will not hold. From 'he had the ability' we cannot infer 'he would have been fully able if he had had the opportunity' since, if t were absent, the premise would be true and the conclusion false.

If Austin is prepared to identify 'he can do X, if he has the ability' with 'he has the opportunity to do X', which at one point in his argument he actually does, it must be because he thinks (in my view rightly) that there is no further relevant sense, t . But if this and the corresponding identification of 'he can do X,

if he has the opportunity' with 'he has the ability to do X' are correct, not only will the restricted categoricals follow from the relevant hypotheticals; we can also conclude that the all-in 'can' is a conjunction of ability and opportunity.⁵

Nevertheless, though ability and opportunity do add up to make the all-in 'can', it would be wrong to say that the all-in 'can' is a *mere* conjunction of ability and opportunity; for these two are themselves conceptually related. Let us consider first what it is to have an opportunity. To say that someone has, here and now, an opportunity for doing X is not to say how things are in the world, though, to be sure, if you know about the activity X, you can deduce much about how things are. It is rather to say that the stage is *so* set that anyone who has, in general, the ability to do X is fully able to do it here and now. For example, to say that he was in a position to smash that lobe is not to describe, even in outline, the relative positions of marble ball and net and the posture of the man. It says rather that marble ball and net were *so* placed in relation to each other and that the man's limbs were *so* disposed that if he has, in general, the ability to smash lobes, he could have smashed that one. But surely, it might be objected, the ability of the player makes no difference to the question whether there was an opportunity or not. Can we not conclude from the relevant statements about the positions of the objects concerned that there certainly was an opportunity to smash a lobe, whether or not the player had the ability to smash it? Certainly, if I know about tennis, I can. But to describe the situation as an *opportunity* is to do more than merely to describe it; it is to say that the conditions for exercising a certain ability

⁵ If 'he has the ability' is equivalent to 'he is fully able, if he has the opportunity' and 'he has the opportunity' is equivalent to 'he is fully able, if he has the ability', it follows truth-functionally that 'he is fully able' is equivalent to 'he has ability and he has opportunity'. Mr. P. T. Geach has pointed out to me that this conclusion can be reached without relying on the rules for the 'if' of material implication, which is certainly an unreliable procedure in an area in which 'ifs' are as slippery as they are here. We need rely only on the principle ' $s \supset t . v \supset w : \supset : s . t \supset v . w$ ' which is valid for a greater variety of 'ifs', including the counter-factual 'if'.

obtained. The description of the spatial relations remains true whether or not the player has any ability; but to see these relations as constituting an opportunity is to see them *as* conditions for the exercise of an ability. An opportunity is essentially an opportunity *for* someone who has an ability (which is why we can know exactly how we are placed without seeing in our situation an opportunity). If this is right, the concept of opportunity presupposes that of ability in the sense that, if there were no abilities, if everything in the world remained exactly as it is, nothing would constitute an opportunity.

In a somewhat different way the concept of ability includes that of opportunity. For an ability to do something is always the ability to do that thing in certain conditions without which the ability could not (logically) exist. Thus, if there are four men in a room where there are no cards, it may be that they can all play bridge in the sense that they have learnt and not forgotten. They cannot play bridge here and now; but this does not mean that their ability has deserted them, to return again when some cards arrive. It is rather that the ability they have is the ability to play-bridge-if-there-are-cards, not the remarkable ability to play-bridge-if-there-are-no-cards. The presence of cards, three other players etc: may be collectively described as constituting an opportunity to play bridge; so that when we say of someone that he has the ability to play bridge we are saying that he has it *when* these conditions are fulfilled. It does *not* follow that when we assert ability we also assert or imply that the conditions *are fulfilled* here and now; the conditions are included in the description of the ability; for the ability is to do the thing in these conditions. It is the fact that an ability is always an ability to do-X-given-an-opportunity that makes it inevitable that, if we have any such concept as 'can' at all, we should have the triple concept of ability, opportunity and both together.

If this account of the relations between the three senses is correct in outline, Austin's picture of the 'all-in' use as the "paradigm use, around which cluster and from which divagate, little by little and along different paths, a whole series of other uses" is radically misleading. In the first place it is not the all-in, but

the ability sense that is fundamental.⁶ But I would not put too much emphasis on this point; it is not so much a question of getting one fundamental sense clear at the start and then seeing how other senses cluster round it, as of getting all three senses clear together by seeing how they fit in with each other. It is the word 'paradigm' that, as so often, gives a false impression; for it suggests that the all-in use provides a pattern or model to which the other uses, to a greater or lesser extent, conform. But, whatever the relations between the all-in and the other uses, it is not paradigmatic of them. The unravelling of a complex concept consists in showing how the elements that make it up (for example the different words which have the same stem, the different senses of a word, the different constructions into which it can enter) are related to each other. In particular we want to know which relations are logically necessary, which elements could not exist without some other. There is no doubt a good explanation for the fact that we use 'foot' both for a measure of length and for a part of the body; but, not only could we have different words for these two concepts, we could have either concept without the other. With 'can' the position is different; for we could not have either of the concepts of ability or opportunity unless we had both. Here we have to do, not with two related concepts, but with one complex concept. In a situation such as this it may be useful to exhibit one or more elements in the complex as fundamental, as giving the key to the others; but it is seldom or never the case that these key elements are paradigmatic of the others. To put the relation in this way is to suggest that, though they cannot be understood without it, it could be understood without them.

⁶ As the etymological connection with 'know' would lead us to expect. Austin finds the same three senses in the case of 'know'; but while there is an ability-sense of 'know' we cannot use 'know' to assert opportunity without ability. A man cannot be said to know Chinese merely on the strength of his living in China and hence having many opportunities to speak it; nor, to take Austin's example, could he be said to know what the thing in my hand is if he has an opportunity to identify it but lacks the ability.

II

So far I have been concerned with the different senses of 'can'. There is no doubt whatever that Austin is right in saying both that this verb is used to make categorical statements and that 'could have', being sometimes its past indicative tense, is also used to make categorical statements. In so far as the idea that 'can' requires analysis in terms of 'if' stems from the idea that 'could have' is subjunctive it is wholly mistaken. But I shall argue in this section that the central 'can' of ability is radically hypothetical, in all its moods and tenses, not in the sense that it must always be escorted by a spoken or unspoken 'if', but in the sense that it can be analysed in terms of 'if'.

I shall be concerned, in this section, solely with that use of 'can' which is relevant in connection with the freedom of the will, to the ascription of responsibility and to moral condemnation. In such cases we are always concerned, not with the question whether anyone can, in general, do or avoid something, but with the question whether he could or could not have done or avoided doing some particular thing that, as a matter of fact, he failed to do or did. This feature gives 'could have' a counterfactual air which tempts us to treat it as a past subjunctive, a temptation into which I certainly fell in *Ethics* with, as Austin shows, disastrous results. When we say, in preparation for an accusation of some kind 'he could have avoided doing that', we are not saying anything conditional or subjunctive. We are saying categorically that he was both able and in a position to avoid doing what he did, and this is shown by the fact that if it can be shown that, as a matter of fact, he lacked either the ability or the opportunity, the accusation fails. Nevertheless it does not follow from the fact that 'could have' is a past indicative and used to make categorical assertions that these assertions may not require analysis in hypothetical terms; for it may be that the whole verb 'can' is susceptible of, or indeed requires analysis in terms of 'does . . . , if . . . '. Abilities are a sort of dispositional, and the idea of a dispositional does not seem to me, as it does to Austin, too obscure to be helpful. 'That chair is unstable' is categorical enough, compared with 'that chair would be unstable if one of

its legs were an inch shorter than it is'; but it does seem to mean something like 'that chair would actually fall over, if p '. To be sure, when we call a chair unstable, we do not say precisely what it would do in precisely what conditions; we cannot spell out the analysis. But this only shows that the concept of instability in ordinary speech is inherently imprecise. Dispositional statements are used to connect an ill-defined, but not limitless set of occurrences with an ill-defined, but not limitless set of conditions.

Of all attempts to analyse 'can' in terms of what actually does or would happen, if something, Austin allows the greatest plausibility to 'he succeeds, if he tries'. This will certainly not do in all cases, since there are things that we can do without trying and in such cases 'trying' and 'success' are not in point. Neither this nor any other single analysis of all 'can'-statements is correct, but, I suggest, some analysis of this kind will always be found to work. I shall, however, limit my discussion to an example which Austin gives in connection with 'I shall succeed in doing X, if I try' as an analysis of 'I can do X'. The example is that of a golfer who misses a short putt, fails, and kicks himself because, things being exactly as they were, he could have holed it. "It is not", he says, "that I should have holed it if conditions had been different: that might of course be so, but I am talking about conditions as they precisely were, and asserting that I could have holed it. There's the rub. Nor does 'I can hole it this time' mean that I shall hole it this time if I try or if anything else: for I may try and miss, and yet not be convinced that I couldn't have done it; indeed further experiments may confirm my belief that I could have done it that time although I didn't. But if I tried my hardest, say, and missed, surely there *must* have been *something* that caused me to fail, that made me unable to succeed? So that I *could not* have holed it. Well, a modern belief in science, in there being an explanation of everything, may make us assent to this argument. But such a belief is not in line with the traditional beliefs enshrined in the word *can*: according to them, a human ability or power or capacity is inherently liable not to produce success, on occasion, and that for no reason (or are bad luck and bad form

sometimes reasons?)." Since this brings us to the edge of the determinist controversy, I propose to examine this example in some detail.

I. Austin notes that the facts alleged conflict with determinism but are in line with the traditional beliefs enshrined in the word 'can'. 'Determinism' he regards as the name for "nothing clear"; but it can be made clear enough for our purposes. For Austin, an ability is something that is inherently liable not to produce success, on occasion, and that for *no* reason; so we may, with sufficient clarity, say that a determinist is one who maintains that if an ability fails to produce success on some occasion there *must* have been a reason. Such a determinist has several lines of reply.

a) He might say that Austin's indeterminist belief is not, in fact, enshrined in our use of 'can'; ordinary language is quite non-committal on this point. Common sense, he will say, has a healthy habit of not coming down on one side or the other in a theoretical controversy which is not in practice decidable. If there *was* a reason for the failure of the ability on that occasion, we do not know it and have no hope of discovering it; so we shrug it off with the phrase 'it was just one of those things'. But it would be quite wrong to suppose that the use of this phrase implies a belief either that 'those things' have causes or that they do not; on this point traditional belief is silent.

b) He might argue that if our ordinary use of 'can' really does enshrine an indeterminist belief, so much the worse for it. You might as well argue that the heliocentric hypothesis is false on the grounds that it conflicts with the traditional beliefs enshrined in the phrase 'terra firma'. To be sure, he must now give up the claim that he is only analysing 'can' as it is traditionally used. But this would not trouble him much. Our actual use of 'can', he will now say, enshrines a false belief, and the sooner we get rid of it the better.

II. To accept the possibility of Austin's example and the interpretation he puts on it is to reject the thesis that 'he can' in this sort of case means 'he *always* does, if . . .'; but it invites, and I shall try to show that it requires, the thesis that 'he can' means

⁷ *op. cit.* pp. 119-20 n.

'he *usually* does, if . . .'. This would be a crucial, indeed a fatal change, if the issue were between determinism and indeterminism; but it is not; the issue is the possibility of an hypothetical analysis of 'can'. Let us agree for the moment that an ability is inherently liable not to produce success, on occasion, and that for no reason. It is also inherently liable to produce *success*, at least on most occasions. If you miss an occasional three-foot putt your ability to hole three-foot putts may not be in doubt; but if you miss too many, you lack the ability. 'He can', then, implies 'he usually succeeds, if he tries'; and equally, though one success might be a fluke, 'he usually succeeds, if he tries' implies 'he can'. One might object that 'he usually succeeds, if he tries' is a much more guarded remark than 'he can', expressing some doubt as to whether he can or not. But since 100 % success is not, on Austin's view, required for the assertion of 'he can', something less than 100 % success must be sufficient if the assertion is ever to be made at all. There is, therefore, at least a material equivalence between 'he can' and 'he usually succeeds, if he tries' in this sort of case.

III. But, if there is a material equivalence, can we go further and assert a connection of meaning? It would certainly be odd to suggest that abilities are *causally* connected with successes.

It may well be that all human abilities are causally dependent on the possession of suitable muscles, nerves, brain cells and the like, and we learn from experience what equipment of this sort is required for each ability; but we do not learn from experience that if a man has a certain ability he will usually succeed when he tries or that he will not usually succeed if he lacks the ability. Austin talks of an ability as *producing* success or failure, as if it were a tool or a part of the body with which we do something and without which we cannot (contingently) do it. The metaphor of 'producing' can hardly be seriously intended; but the point is crucial. We can identify a niblick and then go on to say that it is used for getting out of bunkers; but we cannot identify the ability to get out of bunkers and then establish a contingent, but almost universal correlation between having this ability and actually getting out of bunkers. We do not, in short, learn by

experience that people succeed in doing something if and only if they have the ability to do it.

Consider a contrasting case. I am looking out of the window and see that it is raining hard; I turn away for a moment and when I look back it is still raining. It would be ludicrous to suggest that it was not raining while my back was turned; for we know that rain is not that sort of thing. But, though ludicrous, the suggestion is not unintelligible, not logically absurd. Rain before and rain after is very good evidence; but it is not conclusive evidence. This is quite different from Austin's case, since in his case a run of successes before and after the one failure *conclusively* establishes the truth of the statement that he had the ability even on the occasion on which he failed. Why does Austin's golfer only say that further experiments may *confirm* his belief that he could have holed that putt? They prove it up to the hilt, since, given that n is large enough, a run of $n-1$ successes *entails* the presence of an ability throughout the run. We are not arguing, as we are in the rain case, on the inductive grounds that rain is known to be the sort of thing that does not stop and start when people turn round. Abilities are by definition a sort of things that cannot suddenly stop and start, because they continue through periods of time in spite of failures; so the relation of evidence to conclusion in Austin's case is, like that of the evidence in the rain case to the conclusion 'the weather was foul the whole time I was in the room', one of entailment.

But, it might be asked, might he not have retained his ability to hole putts of that sort throughout half an hour except at the one crucial moment? Not only did he fail, as we know; perhaps also he lacked the ability just then and there? It is important to see that this suggestion is ruled out by Austin's account, not as false, but as unintelligible. Statements of ability are statements to the effect that someone is usually, or would usually be successful in a series of attempts; and what could be meant by saying 'he was usually successful just then and there'? An ability endures through a period and can only be said to exist *at* a time in the sense that everything that endures may be said to exist at every time durings its span of endurance. We may, then, say that

he had the ability just then and there only in the sense that the time referred to by 'then and there' was within this span; and the evidence for his having had the ability just then and there in *this* sense is conclusive.

If Austin's indeterminism (abilities are liable not to produce success, on occasion, and that for no reason) is allowed, why should we not also allow this alternative account of the matter? Abilities, we say, are inherently liable to *desert* us, on occasion, and that for no reason. I lose my ability when I am drunk or tired or in a panic, and sometimes I *just* lose it for no reason at all. If both kinds of indeterminism were allowed, we should have to say that it was an open question which was the correct story to tell on some particular occasion; for the stories are critically different. It might be that he *could* have done it – this being one of the rare occasions on which his ability did not produce success; or it might be that he could *not* have done it – this being one of the rare occasions on which his ability deserted him. But it is clear that we cannot allow this choice of 'explanations'; for since, *ex hypothesi*, there is *no* difference between the two cases, we should never have any reason for preferring one to the other.

I conclude, then, that so far from being a reason for rejecting the possibility of an hypothetical analysis of 'can', Austin's account of the case of the golfer actually confirms it. For, unless the idea of an ability *producing* success is taken more seriously than he intended – and I have given reasons for saying that it cannot – there is a logical equivalence, in this type of case, between 'he can' and 'he succeeds if he tries'. It is true that, if we accept Austin's account, we must rewrite the analysis as 'he usually succeeds, if he tries'; but this, while an important change, leaves the hypothetical analysis intact. One caution is necessary here. Philosophers who have offered analyses of this kind have often written as if they supposed that there was a *causal* connection between the consequent 'he does, or will, or succeeds' and the antecedent 'if he wants to, or chooses, or tries'. This is certainly incorrect.⁸ There is a large number of verbs, including all

⁸ On this point see Richard Taylor: 'I can' in the *Philosophical Review*, January 1960. pp. 82-6.

those mentioned above and many others, which belong to the general area of *intentional action*. Many of the connections between them are, like that between ability and success, certainly not causal but, unlike that between ability and success, not logical either. What their status is I do not know; to find out would be to get to the heart of the matter.

IV

Austin was not, in his paper, directly concerned with questions of freedom and responsibility; but this is an area in which questions of the form 'could he have done it?' typically come up for an answer, and he touches on it when he says that the golfer kicks himself for missing the putt. There are cases in which everything else is beyond reasonable doubt and the question whether or not an ascription of responsibility or some accusation is called for turns solely on the question 'could he have done it?' or 'could he have avoided doing it?' No solution to the second-order philosophical question 'what do statements made with 'can' and 'could have' mean in these contexts?' can be considered satisfactory unless it succeeds in showing why the first-order questions should be considered relevant to ascriptions and accusations. The fact that they so obviously *are* relevant has sometimes blinded philosophers to the difficulty of seeing why this should be so and hence into giving theories of 'can' which will not explain it. This is the rock on which many theories founder. They give us senses of 'can' and 'could have' which may well be correct for other uses of these protean words, but which are such that they cannot explain why 'could have' questions are thought relevant to ascriptions and accusations. This is typically the case with theories of an intuitionist or introspectionist type, according to which both the meaning of 'I can' and the truth (on at least some occasions) of statements in the form 'I can' are held to be transparent.

Austin's theory would be immune from this criticism if it were not for the fact that the golfer reproaches himself for his failure. But it is not clear why he should do this, rather than cursing his luck. Certainly it was *he*, and not one of his team-mates, who

missed the putt and, perhaps, thereby lost the match; and certainly he might lament the fact that his prowess was not greater than in fact it was. But, by the terms of the case, having tried as hard as he could and failed, he has nothing for which to reproach himself. We must distinguish (as philosophers are prone not to do) between the question 'Is determinism true?' and the question 'What, if any, is the bearing of departures, if any, from strict determinism on the freedom of the will, in that sense of freedom which is thought to be a necessary condition of responsibility?'. If we accept the terms of Austin's case, we must give a negative answer to the first question; but if we have no criterion for distinguishing cases in which an ability inexplicably fails to produce success (he could have done it, but did not) from cases in which the ability inexplicably lapses (he could not have done it), the indeterminism which we must accept will have no bearing whatever on freedom. The type of theory that I was criticising in *Ethics* would say that, given the terms of Austin's case, it remained an open question whether or not the golfer could have holed the putt and hence whether or not he could be blamed for the consequent loss of the match. On Austin's view, as we have seen, the first question is no longer open, the evidence being sufficient to entail that he could have holed it; but we are obliged to construe 'he could have holed it' in such a way as to make the man as blameless for the loss of the match as he would have been if he had been unable to hole it. He is, of course, responsible for losing the match in the trivial sense already noticed, that it was he who missed the vital putt; but he would have been equally responsible in this sense if he had missed it through lack of ability. Even if Austin is right in giving an indeterminist account of abilities, he fails to show how and why ascriptions of responsibility may turn on the answers that we give to 'could have' questions.

In Austin's example as it stands no question of moral responsibility or moral appraisal comes up. How could it be made to come up? The player might have been bribed by the other side and have missed the putt on purpose, while pretending to try his hardest. Or, knowing that it was a critical shot in an important

match, he may have been culpably negligent in its execution. (Negligence of this sort, thoughtlessness, lack of consideration, failure to see something in a situation that is morally relevant – all these, though different, belong in the same bag, and it is a bag which moral philosophers have not examined as carefully as they should; for nine tenths of our moral shortcomings will be found in it.) But, to make moral assessment relevant at all, *something* of this kind must be introduced into the situation. I suggest that when, in preparation for a judicial or moral appraisal, we ask ‘could he have helped it?’ what we want to know is the answer to such questions as ‘why did he do it?’, ‘why did he not succeed?’, ‘what prevented him?’, ‘but for what factor in the situation would he have done it?’. Some answers, notably lack of ability and lack of opportunity, exculpate, if not wholly at least to some degree; these are the answers that we summarise in the blanket phrase ‘he could not help it’; other answers, notably ‘he was bribed’ and ‘he just didn’t care’ have the opposite effect. The problem of the freedom of the will, in so far as this is not only a metaphysical problem but relevant to moral philosophy, is that of discerning some general feature that distinguishes the exculpatory from the non-exculpatory answers. To say that some things are in our power to do or not do and others not, though perhaps true, is wholly unenlightening. It merely restates the problem; and theories about what it is in our power to do cannot be acceptable unless they succeed in explaining why the question wheather something is or is not in our power should be considered relevant to our moral status.

An improved proof procedure¹

by

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PART I

Introduction

I will here deal with effective procedures for finding a proof of any valid sequent

$$(1) \quad \Gamma \rightarrow \Theta$$

where Γ and Θ are finite sequences of closed well-formed formulas in the predicate (functional) calculus of first order.² The existence of such an effective procedure was first shown by Skolem [20] and [21]. Here, we will be concerned with the problem of constructing procedures that could be used in practice.³

¹ This work is partly included in a project sponsored by Statens tekniska forskningsråd (Sweden). – I am indebted to professor Wedberg, docent Kanger, fil. lic. Berg, and Mr Voghera for reading the manuscript and making valuable suggestions (see also n. 11).

² (1) can be read: if all the formulas of Γ hold, then some formula of Θ holds. Let $\Gamma_{\&}$ be the conjunction of the formulas of Γ and Θ_v , the disjunction of the formulas of Θ . Then, (1) is equivalent to the formula $\Gamma_{\&} \supset \Theta_v$, or the formula $\sim \Gamma_{\&}$ if Θ is empty, or the formula Θ_v if Γ is empty (I will always suppose that one of Γ and Θ is not empty). – I will usually use expressions autonomously in the sequel.

³ The existence of an effective procedure for proving formulas in the predicate calculus also follows from Gödel's completeness theorem for the predicate calculus (which states that the theorems and the valid formulas of the predicate calculus are the same; the proof of this theorem, however, is built on the Skolem's ideas), since it is always possible to enumerate the expressions that are derivations effectively. (That kind of enumeration procedures is known as "the British museum method" or the "fifty million monkeys technique" from someone's remark that, given long enough, an army of monkeys would be able to type out all the books in the British museum. The

Skolem also showed that the demonstration proving that a sequent is valid can be given in a certain *normal form*, often called Herbrand's normal form; the statement of this fact has later been known as *Herbrand's theorem*. The different effective proof procedures for the predicate calculus of first order described in the literature are all built on variants of Herbrand's theorem and consist simply in generating certain expressions (namely, the *saf:s* of the formula to be proved as defined below) until one is found which is a proof (in normal form) of the formula to be proved. However, if such a procedure should be usable in practice, it has to be very discriminating in generating expressions. Rather than generating expressions until the desired one is found, the procedure should by calculations determine the desired proof. An attempt towards constructing such a procedure will be made here.

I will start with discussing Herbrand's theorem and some generalizations of it in § 1. The proof method proposed in this paper is introduced by some examples in § 2 and is stated more exactly in § 3. In §§ 4 and 5 the merits of the method and the possibility of having a machine to execute the procedure is discussed. These sections (1-5) constitute the first part of the paper. Part II contains a demonstration (built on Herbrand's theorem) of the soundness and completeness of the method (§ 6), some possible modifications of the method (§ 7), and an example of how the method sometimes can be used to show that no proof of a formula is possible (§ 8) (the example is essentially Herbrand's case of the decision problem).

§ 1.1. *Herbrand's theorem*

I will use the following version of *Herbrand's theorem* as starting point:⁴

first to perform such procedures mechanically seems to have been R. Lull with his machine *Ars Magna* in the fourteenth century; see Bowden [4] p. 317.)

⁴ Herbrand's theorem can be found e.g. in Hilbert and Bernays [9] and Dreben [6] in somewhat different versions from the one given above (cf. § 7.5 n. 24). For other bibliographic references concerning theorems related to Herbrand's see § 1.3. (In its original formulation, Herbrand's theorem speaks about provability instead of validity.)

If A is a formula in prenex normal form, then the sequent $\rightarrow A$ is valid if and only if it is demonstrable from a valid (tautological) sequent of the propositional calculus by applications of the following three rules of proof.

$$\begin{array}{l} \text{PR 1.} \quad \frac{\rightarrow \Theta, F(c/x), \Theta'^5}{\rightarrow \Theta, UxF, \Theta'} \end{array}$$

where c does not occur in the sequent below the line.

$$\begin{array}{l} \text{PR 2.} \quad \frac{\rightarrow \Theta, F(c/x), \Theta'}{\rightarrow \Theta, ExF, \Theta'} \end{array}$$

$$\begin{array}{l} \text{PR 3.} \quad \frac{\rightarrow \Theta, ExF, \Theta', ExF, \Theta''^6}{\rightarrow \Theta, ExF, \Theta', \Theta''} \end{array}$$

Let us introduce three *rules of transformation*, TR 1–3, where TR i ($i=1, 2, 3$) is the rule of transforming the sequent below the line into the sequent above the line in PR i (observing the restriction on c in PR 1); thus, a reverse application of PR i . Let us then define a *sequent of associated formulas of A* (abbreviated: *saf of A*) as a sequent of quantifier-free formulas that can be obtained from $\rightarrow A$ by a series of applications of TR 1–3. A saf of A can thus be written in the form

$$\rightarrow A_1, A_2, \dots, A_n$$

where A_i ($i=1, \dots, n$) – which we call an associated formula of A – is obtainable from A by deleting all the quantifiers and substituting constants for the variables.

From Herbrand's theorem we can derive:

- (2) If A is in prenex normal form, then the sequent $\rightarrow A$ is valid if and only if there exists a valid (tautological) saf of A .

⁵ c and x are used to denote individual constants and variables respectively. The notation $F(a/b)$ where a and b are individual variables or constants, is used to denote the result of substituting an occurrence of a for every free occurrence of b in F . The rule is to be understood as stating that the sequent below the line follows from the one above the line.

⁶ PR 1 and 3, as distinguished from PR 2, also hold in the reverse direction i.e. the premiss follows from the conclusion. This fact is essential for the proof procedures we are to consider.

It is clear that (2) above affords an effective proof method, because if A is in prenex normal form, the following holds

(a) we can enumerate the saf:s of A ,

(b) for every saf of A we can decide by propositional calculus whether it is valid or not,

(c) thus, by searching through the saf:s of A testing whether they are valid, we will sooner or later find a valid one, if A is valid,

(d) from a valid saf of A we easily give a proof of $\rightarrow A$ by using the rules of proof PR 1-3 (we may of course omit this last step by simply referring to (2) saying that A is valid if there is a valid saf of A).

To prove a valid sequent $\rightarrow A$, we thus simply generate the saf:s of A in some appropriate order by making different series of applications of the transformation rules TR 1-3 until a valid saf of A is found. In this paper we will consider the problem of so applying the transformation rules that a valid saf of A is found in an expeditious manner.

There are three aspects of this problem corresponding to the following three respects in which the saf:s of A - or rather the series of applications of the transformation rules made in order to obtain the saf:s of A - may differ: They may differ in respect to

(i) the constants substituted at the applications of TR 1 and TR 2,

(ii) which formulas TR 3 has been applied to and the number of such applications, and

(iii) the relative order in which the transformation rules have been applied.

The method to be proposed will determine how the transformation rules shall be applied to a formula A in respect to (i)-(iii) above in order to give a valid saf of A (if such a saf of A exists). Previous proof procedures have found the valid saf of A by trying different combinations of the transformation rules in some arbitrarily defined order until one eventually is found that gives a valid saf of A .

Let us examine the points (i)-(iii) above. First we note that the validity is preserved at the applications of TR 1 and TR 3

(cf. n. 6). The applications of TR 1 that have to be made in order to obtain a saf of A are determined by the quantifiers of A and the applications of TR 3, and can therefore be considered unproblematic as long as we take care to introduce a new constant when replacing the variable.

However, when applying TR 2, it is essential what constant is substituted if validity is to be preserved. In previous proof methods the appropriate constants are found by trying them in some arbitrarily defined order (e.g. an alphabetic one c_1, c_2, \dots) until an appropriate constant is found. The constants tried in this manner may be restricted to the constants already occurring in the sequent to which TR 2 is to be applied.⁸ Suppose that there are m constants occurring in a sequent $\rightarrow A$ and that we have to make n applications of TR 2 to obtain a saf of A . Then there are m^n different combinations of applications of TR 2 giving different saf:s of A , one or a few of which may be valid. As m and n grow with increasing complexity of A , the number m^n of different combinations of applications of TR 2 among which we have to search in order to find an appropriate one soon gets intolerably great. Therefore, previous proof methods can in practice be used to prove only rather simple theorems (cf. § 4 and § 5).⁹

It may not be possible to preserve the validity when applying TR 2 without first applying TR 3. Hence, there is also the problem (ii) above of determining to which formulas TR 3 has to be applied and the number of such applications necessary to obtain a valid saf of A . This problem will further be discussed in § 1.2.

Finally, because of the fact that we have to introduce a new constant when making substitutions at an application of TR 1, the saf:s of A are also dependent on the relative order in which the applications of the transformation rules are made. The order between the applications of TR 1 and 2 made to *one* associated for-

⁸ This restriction is made in the proof procedures developed by Beth [2], Hintikka [10], Kanger [11], Quine [18], and Schütte [19].

⁹ The intention of the scholars who have developed proof methods has not only been to give a method for finding derivations but has often primarily been to facilitate the presentation of the completeness proof for the predicate calculus.

mula in a saf of A is of course determined by the order in which the quantifiers stand in A . In the proof method to be proposed the order between the different applications of the transformation rules will conveniently be determined when the order matters and is not determined in the way just described.

§ 1.2. *Applications of TR 3*

To obtain a valid saf of A , we usually have to apply TR 3 to some formulas a certain number of times. Let us consider two methods for finding the appropriate applications of TR 3 in order to obtain a valid saf of A .

The *first method* is the one used in all previous proof procedures that contain a method for enumerating the saf:s of A . The saf:s of A are formed in an order $S_1, S_2, \dots, S_i, \dots$, where S_1 contains one formula and S_{i+1} is similar to S_i but is obtained by one more applications of TR 3 (and necessary applications of TR 1 and 2 following this application of TR 3). If A is valid, we finally obtain a valid sequent S_n in this way provided that we vary the constants substituted at applications of TR 2 and the formulas to which we apply TR 3. The sequents in the list S_1, S_2, \dots that we have to test for validity, will however usually be unmanageably long before we reach the valid sequent S_n (because of all the associated formulas obtained by inappropriate applications of TR 2 not contributing to making S_n tautological).

The number of saf:s of A that differ in respect to the properties (i)–(iii) listed above in § 1 is finite for every fixed number of applications of TR 3. Thus there is a *second method* in which we first form all the saf:s of A obtainable by no application of TR 3, then all the saf:s of A obtainable by one application of TR 3, then all the saf:s of A obtainable by two applications of TR 3, and so on. If A is valid, we finally reach the smallest valid saf of A in this way (i.e. the valid saf of A that contains the least number of formulas; the number of formulas of a saf of A is obviously one greater than the number of applications of TR 3 made in order to obtain the saf).

The method to be proposed here will like the second method

find the smallest valid *saf* of A , but it will not be necessary to consider all the *saf*s of A obtainable by a given number of applications of TR 3. Furthermore, it will also be possible to determine what formulas TR 3 is to be applied to without first having to try the different possibilities.

§ 1.3. Existing proof methods

The proof method obtained from Herbrand's theorem has been improved in some respects (besides the improvement of restricting the constants substituted at applications of TR 2, see n. 8). Gentzen [7] extended it to hold for not only one but a number of formulas written in the sequent form like (1) (the same result was later also obtained by Quine [18]). We cover this extension in *Gentzen's theorem* by slightly modifying Herbrand's theorem as formulated in § 1.1:

If the formulas of Γ and Θ are in prenex normal form, then the sequent $\Gamma \rightarrow \Theta$ is valid if and only if it is provable from a tautological sequent by using the following six rules of proof.

Three rules, PR ia ($i=1, 2, 3$), are like PR i in Herbrand's theorem except that we add Γ to the antecedents of the sequents that occur in these rules. The other three rules, PR ib ($i=1, 2, 3$) are the duals of PR ia ; namely

$$\begin{array}{l} \text{PR 1 b.} \quad \Gamma, F(c/x), \Gamma' \rightarrow \Theta \\ \quad \quad \quad \Gamma, \text{Ex}F, \Gamma' \rightarrow \Theta \end{array}$$

where c does not occur in the sequent below the line.

$$\begin{array}{l} \text{PR 2 b.} \quad \Gamma, F(c/x), \Gamma' \rightarrow \Theta \\ \quad \quad \quad \Gamma, \text{Ux}F, \Gamma' \rightarrow \Theta \end{array}$$

$$\begin{array}{l} \text{PR 3 b.} \quad \Gamma, \text{Ux}F, \Gamma', \text{Ux}F, \Gamma'' \rightarrow \Theta \\ \quad \quad \quad \Gamma, \text{Ux}F, \Gamma', \Gamma'' \rightarrow \Theta \end{array}$$

Let TR ia and b ($i=1, 2, 3$) be the reverse application of PR ia and b like before.

Gentzen's result, in turn, was generalized to hold for formulas also not in prenex normal form by Beth [2], Hintikka [10],

Kanger [11], and Schütte [19]. To cover this extension, we have to generalize the rules of proof to cover also formulas with quantifiers not initially placed (cf. § 7.2). If we introduce rules of transformation corresponding to such rules of proof and speak about a saf of a sequent $\Gamma \rightarrow \Theta$ accordingly, we obtain the following more general version of (2):

The sequent $\Gamma \rightarrow \Theta$ is valid if and only if there exist a valid (tautological) saf of $\Gamma \rightarrow \Theta$.

§ 2. A first outline of the proof method

Before getting into technical details in § 3 let us consider some examples to get an intuitive picture of the proof method that is to be developed. The proof of the following sequent was discussed by Beth [3]:

$$\text{UxUy}(Px \vee Qy) \rightarrow (\text{UxPx} \vee \text{UyQy})$$

Let us write this sequent in the equivalent form

$$(4) \quad \text{UxUy}(Px \vee Qy) \rightarrow \text{UxPx}, \text{UyQy}$$

To prove (4) we now try to find a tautological saf of (4). First we apply TR 1 a twice and obtain the sequent

$$(5) \quad \text{UxUy}(Px \vee Qy) \rightarrow Pc_1, Qc_2$$

We have now to make two applications of TR 2b. If we only consider substitutions of c_1 and c_2 , there are four different ways in which the two applications of TR 2b can be made. If we substitute c_1 for both x and y we obtain

$$(Pc_1 \vee Qc_1) \rightarrow Pc_1, Qc_2,$$

if we substitute c_1 for x and c_2 for y we obtain

$$(Pc_1 \vee Qc_2) \rightarrow Pc_1, Qc_2,$$

if we substitute c_2 for x and c_1 for y we obtain

$$(Pc_2 \vee Qc_1) \rightarrow Pc_1, Qc_2,$$

and if we substitute c_2 for both x and y we obtain

$$(Pc_2 \vee Qc_2) \rightarrow Pc_1, Qc_2.$$

Only one of these substitutions, namely the one of substituting c_1 for x and c_2 for y , gives a tautological saf of (4).¹⁰ In order to find the appropriate substitutions in a convenient manner, we may first substitute dummies – syntactical variables, say d_1 and d_2 , standing for arbitrary constants – for the variables when we are to apply TR 2a or 2b.¹¹

Applying this device to (5), we get

$$(6) \quad (Pd_1 \vee Qd_2) \rightarrow Pc_1, Qc_2$$

The problem is now to determine the values of d_1 and d_2 so that (6) becomes tautological. A solution is found most easily by re-writing (6) in conjunctive normal form:

$$(7) \quad \rightarrow (\sim Pd_1 \vee Pc_1 \vee Qc_2) \ \& \ (\sim Qd_2 \vee Pc_1 \vee Qc_2)$$

or if we also compound sequents with connectives, we can write

$$(8) \quad (Pd_1 \rightarrow Pc_1, Qc_2) \ \& \ (Qd_2 \rightarrow Pc_1, Qc_2).$$

By just comparing the atomic formulas of each conjunction clause of (7) or (8), we now find that the value $d_1 = c_1$ makes the first clause tautological and that the value $d_2 = c_2$ makes the second clause tautological.¹² This system of the values of the dummies makes

¹⁰ Beth [3] remarks, "This example shows that an intelligent selection . . . (of the substitutions) . . . simplifies the construction. It remains to be seen in how far it would prove convenient to imitate this intelligent way of proceeding in a logical theory machine." He seems to think of finding a solution on the lines of a statistical analysis of which substitutions prove successful on formulas that are decidable. The solution proposed here is quite different but well suited for mechanical use. Cf. the remarks on heuristic methods in n. 22

¹¹ Docent Kanger has much promoted the idea of using dummies in this connection. The use of dummies (in the form of free variables standing for arbitrary constants) was suggested in Prawitz [15], an essay for the seminar for theoretical philosophy at the University of Stockholm in 1957, and was discussed at the seminar. At these discussions docent Kanger advocated and furthered the idea, which will also be included in Kanger [12]. I thank him for stimulating talks on these topics.

¹² This is of course the point: We do not need to try the different values of the dummies to find the values that make the conjunction clauses of (8) (e.g.) tautological. Instead we can just put identity sign between the arguments of two occurrences of the same predicate sign that stand on different sides of the arrow.

thus (7) and (8) tautological and hence also the sequent (6), which becomes a valid *saf* of (4).

To see how the outlined technique works on a more complicated example, let us consider the proof of the following sequent

$$(9) \quad \rightarrow Ux Ey Uz ((Px \supset \sim Pz) \supset \sim Py).$$

Applying TR 1 and substituting c_1 for x , we obtain

$$\rightarrow Ey Uz ((Pc_1 \supset \sim Pz) \supset \sim Py).$$

Substituting a dummy d_1 for y when applying TR 2, we obtain

$$\rightarrow Uz ((Pc_1 \supset \sim Pz) \supset \sim Pd_1)$$

to which we again apply TR 1 substituting c_2 for z and obtaining

$$(10) \quad \rightarrow ((Pc_1 \supset \sim Pc_2) \supset \sim Pd_1).$$

We rewrite this sequent in conjunctive normal form and obtain

$$(11) \quad (Pd_1 \rightarrow Pc_1) \ \& \ (Pd_1 \rightarrow Pc_2)$$

As in the first example, we now find that the first sequent of the conjunction (11) becomes tautological for the value $d_1=c_1$ and that the second sequent becomes tautological for the value $d_1=c_2$. This, however, does not solve our problem to prove (9), because, in the first place, d_1 could clearly not be the constants c_1 and c_2 simultaneously; c_1 and c_2 are supposed to be two distinct constants as they have to be if the second application of TR 1 above shall be correct (*restriction 1*, see below).

In the second place, the value $d_1=c_2$ is not possible. c_2 is substituted at an application of TR 1 that succeeded the application of TR 2 at which d_1 was substituted. Hence, if d_1 was c_2 , then c_2 would already have occurred in the sequent to which TR 1 was applied and the application of TR 1 would thus have been incorrect. (We will express this fact by the inequality $d_1 < c_2$, which could be read " c_2 is introduced later than d_1 " and which thus excludes $d_1=c_2$; *restriction 3*, see below.)

Thus we see that all values of the dummies of a sequent that are obtained by the technique described above, can not be accepted, because – although the values make the sequent tauto-

logical – some of them may not make the sequent a saf of the sequent to be proved. We will therefore put some *restrictions* on the values of the dummies. Two of them, called restriction 1 and restriction 3, are illustrated by the example above.

Continuing with the example above, we find that the only system of dummy values that make (11), and hence also (10), tautological contradicts the restrictions (i.e. does not make (10) a saf of the sequent (9) to be proved). A valid saf of (9) must thus contain at least two formulas, thus requiring at least one application of TR 3.

If we had applied TR 3 before applying TR 2, we would have obtained

$$\rightarrow E y U z ((P_{c_1} \supset \sim P_z) \supset \sim P_y), E y U z ((P_{c_1} \supset \sim P_z) \supset \sim P_y)$$

Applying TR 2 and TR 1 to the first formula of this sequent as before and TR 2 and TR 1 to the second formula in a similar manner now substituting a dummy d_2 for y and a constant c_3 for z , we obtain the sequent

$$(12) \quad \rightarrow ((P_{c_1} \supset \sim P_{c_2}) \supset \sim P_{d_1}), ((P_{c_1} \supset \sim P_{c_3}) \supset \sim P_{d_2})$$

Rewriting this sequent in conjunctive normal form we obtain

$$(13) \quad (P_{d_1}, P_{d_2} \rightarrow P_{c_1}) \ \& \ (P_{d_1}, P_{d_2} \rightarrow P_{c_1}, P_{c_3}) \ \& \\ (P_{d_1}, P_{d_2} \rightarrow P_{c_2}, P_{c_1}) \ \& \ (P_{d_1}, P_{d_2} \rightarrow P_{c_2}, P_{c_3})$$

Every sequent of (13) now becomes tautological for a number of values of the dummies. The first sequent e.g. becomes tautological if $d_1=c_1$ or $d_2=c_1$. The expression $(d_1=c_1 \vee d_2=c_1)$, which states the necessary and sufficient condition of the first sequent being a tautology, is called the *identity condition* associated with the sequent in question. When forming the identity conditions associated with the different sequents of (13), we immediately omit values contradicting restriction 3 (i.e. $d_1=c_2$ and $d_2=c_3$). The conjunction of the identity conditions is then as follows:

$$(d_1=c_1 \vee d_2=c_1) \ \& \ (d_1=c_1 \vee d_1=c_3 \vee d_2=c_1) \ \& \\ (d_1=c_1 \vee d_2=c_2 \vee d_2=c_1) \ \& \ (d_1=c_3 \vee d_2=c_2)$$

Rewriting this conjunction in disjunctive normal form and omitting redundant clauses we obtain

$$(d_1=c_1 \ \& \ d_1=c_3) \vee (d_1=c_1 \ \& \ d_2=c_2) \vee \\ \vee (d_2=c_1 \ \& \ d_1=c_3) \vee (d_2=c_1 \ \& \ d_2=c_2).$$

A conjunction of the so obtained expression represents a system of values of the dummies that makes (13), and hence also (12), tautological. The first and fourth system, however, contradict restriction 1. Only the systems $(d_1=c_1 \ \& \ d_2=c_2)$ and $(d_2=c_1 \ \& \ d_1=c_3)$ make (12) both a tautology and a saf of (9).

To illustrate *restriction 2* we consider the proof of a sequent S of the form

$$S: \quad \rightarrow ExUyEzF$$

where F is a quantifier-free formula. Substituting the dummies d_1 and d_2 for x and z and a constant c_1 for y at the applications of TR 2 and TR 1, we obtain the sequent S_1

$$S_1: \quad \rightarrow F(d_1/x) (c_1/y) (d_2/z).$$

Suppose now that S_1 becomes a tautology for no values of the dummies. Then we have to make an application of TR 3, applying it either to $\rightarrow ExUyEzF$ or to $\rightarrow EzF(d_1/x) (c_1/y)$. To determine which application is the appropriate one, our method will work as follows. We generate a new formula from F by substituting dummies d_3 and d_4 for x and z and a constant c_2 for y , and add it to the sequent S_1 , obtaining the sequent S_2

$$S_2: \quad \rightarrow F(d_1/x) (c_1/y) (d_2/z), F(d_3/x) (c_2/y) (d_4/z).$$

We now associate identity conditions with the conjunction clauses of S_2 transformed to conjunctive normal form as before, and if we obtain a system of dummy values which makes S_2 valid and which contains the identity $c_1=c_2$, then TR 3 was to have been applied to the sequent $\rightarrow EzF(d_1/x) (c_1/y)$. In that case, d_1 has to be equal to d_3 if S_2 is to be a saf of S . This is the content of restriction 2, in this case stating: if $c_1=c_2$, then $d_1=d_3$.

§ 3. The proof method

The proof procedure is carried out in steps. First there are four *preparatory steps*. Then, there are four steps that may be repeated in the same order any number of times. The procedure is thus naturally divided into *cycles*, and the steps are called the *phases* of respective cycle.

We now suppose that we are to prove a sequent $\Gamma \rightarrow \Theta$ comprising the formulas

$$(14) \quad F_1, \dots, F_f, F_{f+1}, \dots, F_g$$

where F_i , $i \leq f$, is a Γ -formula and F_j , $f < j \leq g$, is a Θ -formula. I will speak about the number of a formula thinking of the enumeration (14). Also the results of transforming these formulas will be numbered and divided into Γ - and Θ -formulas in the same way. If individual constants occur in the formulas in (14), we will suppose they are the first n ones in an enumeration c_1, c_2, \dots

§ 3.1. The preparatory steps of the procedure

Preparatory step I. Write every formula in prenex normal form.¹³

Definition I. In the formulas resulting from step I, existential quantifiers (E) in Γ -formulas and universal quantifiers (U) in Θ -formulas are called *C-quantifiers* (constant-generating quantifiers; cf. the condition on c in PR 1 a and b). The other quantifiers, U in Γ -formulas and E in Θ -formulas, are called *D-quantifiers* (dummy-generating quantifiers; cf. the preliminary explanations in § 2).

Preparatory step II. Make the following transformation of every formula resulting from step I: Delete every C-quantifier (with its attached variable) not preceded by a D-quantifier and substitute a constant for all (remaining) occurrences of the quantified variable. We use c_{n+1}, c_{n+2}, \dots in this order as constants; c_1

¹³ The proof procedure will be shortened if we follow certain rules when carrying out this step see § 7.1. I will also consider an alternative method not requiring step I; see § 7.2.

c_2, \dots, c_n are supposed to be the constants that already occur in the formulas (see § 3). We call the constants c_k ($k=1, \dots, n, n+1, \dots$) *simple constants* to distinguish them from other constants introduced below.

Preparatory step III. Make the following transformation of every formula resulting from step II: Delete every C-quantifier (with its attached variable) and substitute a constant $c(p, q, 1)$ for all (remaining) occurrences of the variable. The constant $c(p, q, 1)$ is called a complex constant and is to be such that

p denotes the number of the formula in which the quantifier occurs (cf. (14)),

q denotes the number of the quantifier in the formula, counting the quantifiers from the left to the right.

In the same way delete every D-quantifier (with its attached variable) and substitute a *dummy* $d(p, q, 1)$ for all (remaining) occurrences of the quantified variable.

Preparatory step IV. We now have g quantifier-free formulas

$$(15) \quad F'_1, \dots, F'_f, F'_{f+1}, \dots, F'_g.$$

Form the disjunction of the negations of the F -formulas and the Θ -formulas:

$$(16) \quad (\sim F'_1 \vee \dots \vee \sim F'_f \vee F'_{f+1} \vee \dots \vee F'_g).$$

(16) is called the *origin formula*, abbreviated: OF.

§ 3.2. The phases of a cycle of the procedure

When starting a new cycle we call it the s :th cycle if the just preceding cycle was the $(s-1)$:th cycle. (The first cycle has the number 1.) Let the cycle now to be described be the t :th cycle ($t=1, 2, \dots$).

Phase I. Transform an instance of the origin formula, OF, by substituting t for the third index in all the dummies and complex constants. The resulting formula is called OF_t .

Phase II. Form the t :th cycle formula, abbreviated: CF_t , defined as follows. The first cycle formula is OF_1 transformed to conjunctive normal form. If the s :th cycle formula is CF_s , then

the $(s+1)$:th cycle formula is $(CF_s \vee OF_{s+1})$ transformed to conjunctive normal form. (CF_t is thus the result of transforming the formula $(OF_1 \vee OF_2 \vee \dots \vee OF_t)$ to conjunctive normal form.)

Phase III. (a) If there is a conjunction clause of CF_t in which no predicate symbol (with any number of arguments) occurs both negated and not negated, then the given sequent $\Gamma \rightarrow \Theta$ is not provable and the procedure is concluded. Otherwise we continue and apply (b) below.

Remark: If there is a conjunction clause of CF_t as described in (a) above, then clearly there is such a clause of every CF_s ($s=1, 2, \dots$). Then no saf of the given sequent $\Gamma \rightarrow \Theta$ can be a tautology, and hence, according to Herbrand's (or Gentzen's) theorem, the sequent $\Gamma \rightarrow \Theta$ is not valid. (Cf. e.g. Church [5] p. 180, which states that a formula of the predicate calculus is not valid if its afp is not a tautology.)

(b) We now associate an *identity condition*, IC_i , with every conjunction clause, M_i , of CF_t ($i=1, 2, \dots, m$, where m is the number of conjunction clauses of CF_t), as follows.

(i) If M_i contains two formulas one of which is the negation of the other, then IC_i is $c_1 = c_1$.

(ii) If M_i does not contain two formulas as in (i), then for every couple of formulas $Pa_1a_2 \dots a_k$ and $\sim Pa'_1a'_2 \dots a'_k$ in M_i (P is a predicate symbol with k arguments consisting of constants or dummies) we form an *identity list*, $IL_{i,j}$ ($j=1, 2, \dots, n_i$; n_i is supposed to be the number of such couples occurring in M_i), of the form:

$$(17) \quad (a_1 = a'_1 \ \& \ a_2 = a'_2 \ \& \ \dots \ a_k = a'_k).$$

IC_i is to be the disjunction of all the n_i identity lists:

$$(18) \quad (IL_{i,1} \vee IL_{i,2} \vee \dots \vee IL_{i,n_i})$$

Phase IV. The given sequent $\Gamma \rightarrow \Theta$ is provable in cycle t if and only if the conjunction of the identity conditions

$$(19) \quad (IL_{1,1} \vee IL_{1,2} \vee \dots \vee IL_{1,n_1}) \ \& \ (IL_{2,1} \vee IL_{2,2} \vee \dots \vee IL_{2,n_2}) \ \& \ \dots \\ \& \ (IL_{m,1} \vee IL_{m,2} \vee \dots \vee IL_{m,n_m})$$

(m is supposed to be the number of conjunction clauses of CF_t ,

n_1, n_2, \dots, n_m are as in (18)) does not contradict the following three restrictions. If (19) contradicts the restrictions, we start a new cycle. — Below, k, p, q, s , and x are any positive integers.

Restriction 1. Two different constants are never equal if they are not complex and differ from each other only in respect to the third index. Thus, $c_k \neq c_{k'}$ if $k \neq k'$; $c_k \neq c(p, q, s)$; and $c(p, q, s) \neq c(p', q', s')$ if not $p = p'$ and $q = q'$.

Restriction 2. If $c(p, q, s) = c(p, q, s')$, then for all $x < q$: $d(p, x, s) = d(p, x, s')$.¹⁵

Restriction 3. If $q < q'$, then $d(p, q, s) < c(p, q', s)$.

Discussion. The identities can obviously be considered as formal expressions belonging to a formal system that comprises the ordinary rules for propositional calculus, equality and inequality (such as laws for the symmetry and transitivity of equality) and the additional rules and postulates stated in the restrictions 1–3. The question to be answered in phase IV is then whether we can infer a contradiction ($a = a' \ \& \ a \neq a'$) or ($a < a' \ \& \ a \nless a'$) from (19) in this formal system.

To determine whether (19) contradicts the restriction, we can rewrite (19) in disjunctive normal form. Every disjunction clause

$$(II_{1,j_1} \ \& \ II_{2,j_2} \ \& \ \dots \ \& \ II_{m,j_m}) \\ (j_i = 1, \dots, n_i; i = 1, \dots, m; n_i \text{ as in (18)}; m \text{ as in (19)})$$

of the so obtained expression that does not contradict the restrictions gives a usable system of values of the dummies, i.e. a system that makes all conjunction clauses of CF_i tautological without contradicting the restrictions. In this way we get all such systems of the value dummies.

When determining in practice whether (19) contradicts the restrictions, several short cuts can be used. First, we can omit every identity list that in itself contradicts the restrictions. If all

¹⁵ To simplify the soundness proof (§ 6.1), we give the following formulation of restriction 2:

Restriction 2'. Like restriction 2 except for the consequence to which we add: and $c(p, x, s) = c(p, x, s')$.

It is easily verified that the question whether (19) contradicts the restrictions is not effected if we use restriction 2' instead of restriction 2.

the identity lists of an identity condition are omitted in this way, then the given sequent is of course not provable in the cycle and we have to start another one (cf. also § 8 about not provable formulas).

Second, we can note that the same identity list occurs in a number of identity conditions at regular intervals (similar to the intervals defined in § 7.3). Using this fact when we rewrite (19) in disjunctive normal form, we can obtain a much shorter formula than the one we would get if we just applied the distributive laws of conjunction and disjunction in the ordinary way. As the problem of determining whether (19) contradicts the restrictions belongs to propositional calculus, we may also benefit from the many investigations in this field, e.g. the research in methods for simplifying truth functions to which Nelson [13] and Quine [17] among others have contributed.

Also worth noticing is the fact that it is sufficient to find *one* system of truth values of the truth constituent of (19) that makes (19) true (or, what is the same thing, to find *one* system of dummy values that makes CF_i tautological) but does not contradict the restrictions. We can therefore use the so-called feedback principle. Special logical machines have recently been built to solve just that kind of problem (see Bowden [4], ch. 15, especially pp. 188–192).

§ 4. An example

To demonstrate what we gain with the described proof method, I will consider a very simple example:

$$(20) \quad UxUyUzUuUvUw((Pxyzu \& Pxyzv \& Quvw) \supset Pxyzw) \rightarrow \\ UxUyUzUuUvUw((Pxyzu \& \sim Pxyzv \& Pxyzw) \supset \sim Quvw)^{10}$$

¹⁰ To make the example more interesting, we may interpret $Pxyzu$ as saying that the point u is in the plane determined by the points x , y , and z , and $Qxyz$ as saying that the point z is on the line determined by the points x and y . Then, (20) is the statement that from the premiss that any point (say w) on a line (determined by points u and v) is in any plane (say one determined by points x , y , and z) to which the line belongs, we can infer the consequence that a point not in a plane to which a line belongs, is not on the line.

The two formulas are equivalent as is easily seen. One is got from the other by a tautological transformation of the matrix and a change of the places of v and w only in the matrix.

If we apply the proof method to (20), we first substitute constants c_1, \dots, c_6 for the variables of the second formula (preparatory step II) and substitute dummies $d(1, q, 1)$, where $q=1, \dots, 6$, for the variables in the first formula (preparatory step III). To save space we omit all the indices of the dummies except the q -index and use d_1, \dots, d_6 as dummies. The first cycle formula (phase II) is composed of four conjunction clauses, $M_i, i=1, \dots, 4$. The following four formulas occur in all the conjunction clauses:

$$Pc_1c_2c_3c_4, \sim Pc_1c_2c_3c_5, Pc_1c_2c_3c_6, \text{ and } Qc_4c_6c_5.$$

Besides these formulas M_1 contains the formula $\sim Pd_1d_2d_3d_4$, M_2 , the formula $\sim Pd_1d_2d_3d_5$, M_3 , the formula $\sim Qd_4d_5d_6$, and M_4 , the formula $Pd_1d_2d_3d_6$.

When we form the identity lists (phase III), we can immediately omit the ones that contain an identity between constants (according to restriction 1 in phase IV). We then get the following identity conditions

$$IC_1 : IL_{1,1} \vee IL_{1,2} : (d_1=c_1 \& d_2=c_2 \& d_3=c_3 \& d_4=c_4) \vee (d_1=c_1 \& \\ \& d_2=c_2 \& d_3=c_3 \& d_4=c_6)$$

$$IC_2 : IL_{2,1} \vee IL_{2,2} : (d_1=c_1 \& d_2=c_2 \& d_3=c_3 \& d_5=c_4) \vee (d_1=c_1 \& \\ \& d_2=c_2 \& d_3=c_3 \& d_5=c_6)$$

$$IC_3 : IL_{3,1} : (d_4=c_4 \& d_5=c_6 \& d_6=c_5)$$

$$IC_4 : IL_{4,1} : (d_1=c_1 \& d_2=c_2 \& d_3=c_3 \& d_6=c_5)$$

Every $IL_{i,j}$ above gives a system of dummy values that does not contradict the restrictions but makes M_i a tautology. If we form the conjunction of the identity conditions (like (19) in phase IV) and rewrite it in disjunctive normal form, we find that only one disjunction clause of the so obtained expression does not contradict the restrictions, namely $(IL_{1,1} \& IL_{2,2} \& IL_{3,1} \& IL_{4,1})$; i.e. only the system $d_1=c_1, d_2=c_2, d_3=c_3, d_4=c_4, d_5=c_6, d_6=c_5$ makes all conjunction clauses of CF_1 tautologies without contradicting the restrictions.

If we had used any other proof method available in literature (cf. § 1), we could also have started with introducing six constants for the variables in the second formula. But when making substitutions in the first formula, we would have 6^6 different combinations of the constants to choose between. As we have seen only one of these combinations of substitutions (namely the one of substituting $c_1, c_2, c_3, c_4, c_6, c_5$ in this order) gives a tautological saf of (20). It can also rather easily be shown that there is no valid saf of (20) that contains a number of formulas but not the formula obtained by the substitutions mentioned above. Thus, to find a valid saf of (20), we would have to form between 1 and 6^6 associated formulas of the first formula in (20). The exact number depends on the order in which we form the associated formulas; i.e. the order in which we make the substitutions in the first formula. Since we have no method to choose a favourable order, we reckon $\frac{6^6}{2} = 23,328$ as the mean number of the associated formulas that we have to form.¹⁷ We also have to determine whether

¹⁷ If we use the substitution order that is defined by Dreben [6] or Kanger [11], we have to form 39,842 associated formulas before finding the appropriate one (i.e. the one needed in a saf of (20)). (This number can be calculated by using the fact that the number of n -tuples of positive integers whose sum is less or equal to m , is $\binom{m}{n}$). The order defined by Dreben is such that of two combinations of constants the one comes before whose sum of the indices is less (the constants are supposed to be given in a list c_1, c_2, \dots). Dreben also uses constants with indices greater than those of the constants introduced when replacing C-quantified variables; i.e. in this case indices greater than 6. Kanger uses the constant c_1 in place of such constants, but otherwise his order agrees with Dreben's. Thus, all the associated formulas of the first formula of (20) where the sums of the indices of the constants substituted are less than 21, come before the appropriate associated formula, and there are $\binom{20}{6} = 38,760$ such associated formulas. The combinations of constants where the sums of the indices agree, are ordered in lexicographic order, and there are 1,082 combinations where the sums are 21 and which come before the combination of constants substituted in the tautological associated formula. Thus, in total, there are $38,760 + 1,082 = 39,842$ associated formulas that come before the appropriate one.)

Beth [2] partially defines an order in which we would have to form be

the associated formulas formed are tautologies. Suppose that we use the first of the two methods discussed in § 1.2. We then get a saf of (20) that contains 23,329 formulas. When determining whether this saf is tautological, we get a formula with $4^{23,329}$ conjunction clauses if we use the method of rewriting the formula in conjunctive normal form. (As some conjunction clauses can be shown to be tautological at an early stage before the whole clause has been formed, the number of conjunction clauses can be somewhat diminished. It can however easily be shown that – using Beth's or Kanger's terminology – we get more than $2^{23,315}$ different semantical subtableaux or deduction branches, respectively, when proving (20). Even a fast electronic computer would need more time to carry out such a derivation than the earth is old.)

§ 5. *Logical machines*

The high speed of modern electronic computers makes it natural to attempt to use these machines for handling proof procedures. In the case of the propositional calculus, mechanical methods have been used for a long time.¹⁸ The possibility to perform more extensive logical derivations mechanically, has been discussed several times, e.g. by Beth [2] and [3] (in Beth [3] references can also be found to discussions by Gelernter and Robinson). To realize this possibility for the predicate calculus, a mechanized method in the form of a pseudo-program was developed by D. Prawitz [15], building on Beth [2] and Kanger [11]. A

tween $5^6 = 15,625$ and $6^6 = 46,656$ associated formulas before finding the appropriate one. (In Beth's order all combinations of constants with indices less than n are substituted before any combination of constants containing a constant with the index n .)

¹⁸ The first machine to solve problems in the propositional calculus was constructed by Jevons in 1869. Several logical machines have recently been built, which accomplish about the same thing as Jevon's machine but much faster (due to the development of electronical technique and sometimes due to the use of more subtle logical principles). More numerous than machines designed exclusively for logical problems, are all the existing programs designed to solve various problems in the propositional calculus by ordinary electronic digital computers. For further details see Bowden [5] ch. 15.

program for an electronic digital computer¹⁹ was subsequently prepared by H. Prawitz in 1957 and modified and tested by N. Voghera in 1958.²⁰ A similar program has also been developed by P. C. Gilmore in 1959.²¹ With some limitations pertaining to memory space, these programs are designed to prove any theorem and refute some non-theorems of the predicate calculus. As the programs rely upon the kind of proof procedures discussed in § 1 where one in some arbitrarily defined order has to try the different possibilities of applying the transformation rules to find the appropriate applications, the program can in practice be used to prove only very simple theorems, time being the restrictive factor.

The proof procedure developed in this paper is meant to be realizable in an electronic computer. Such a realization would considerably enlarge the range of theorems mechanically provable.²² Still, many further improvements are necessary to obtain

¹⁹ Facit EDB, AB Åtvidabergs Industrier, Stockholm.

²⁰ An account of the project will appear in Prawitz D., Prawitz H., and Voghera [16].

²¹ Gilmore's work was presented at the International Conference on Information Processing arranged by Unesco in 1959. It was also mentioned at the conference that a similar work by H. Wang is forthcoming. (Added in proof: Since then, Gilmore and Wang have separately published papers on the subject in IBM Journal of Research and Development, vol. 4 (1960). Another program has been suggested but not yet programmed for machine by Davis and Putnam in "A computational proof procedure", Rensselaer Polytechnic Institute, AFOSR TR 59-124, October 1959. This program is superior to the other three in respect to the propositional part but is similar in the respect of being based on the same procedure of exhausting the different possibilities of applying the transformation rules.)

²² Originating with Newell and Simon [14], there is some research in proof methods with a different approach favouring the use of anthropomorphic heuristics more than algorithms. A heuristic is then meant to be a method which sometimes leads astray but often gives short cuts as compared with the complete algorithms. After having remarked that a proof procedure building on an enumeration procedure of the kind described in n. 3 rapidly exhausts the capacity of the machine, Gelernter [8], concludes, "The remaining alternative is to have the machine rely upon heuristic methods".

This conclusion seems to be somewhat hasty (although the research in heuristics definitely seems to be very interesting), as we can not exclude the possibility of finding fast proof algorithms not relying upon enumerations of

a mechanical proof procedure able to solve more interesting problems. (An example of another improvement is a proof method developed by Kanger [12], which comprises rules for equality and signs for operations, such as addition and multiplication. Such rules and signs are advantageous when applying the proof method to most mathematical theories.)

PART II

§ 6.1. *The soundness of the proof method*

To see that the proof method is sound, i.e. that any sequent provable with the method is valid, we make use of Gentzen's theorem as stated in § 1.3 as follows.

Suppose that the given sequent $\Gamma \rightarrow \Theta$ is provable with the proof method in cycle t and suppose that the formulas are in prenex normal form. Then the conjunction of the identity conditions (19) obtained in phase IV of cycle t does not contradict the three restrictions listed in phase IV. Thus there is a conjunction of identity lists of the form

$$(21) \quad (IL_{1,j_1} \& IL_{2,j_2} \& \dots \& IL_{m,j_m})$$

($j_i \leq n_i$; $i=1, \dots, m$; n_i as in (18); m is the number of conjunction clauses of CF_t) that does not contradict the restrictions (cf. the discussion following phase IV in § 3.2). Make the following three transformations of CF_t (the cycle formula obtained in phase II of cycle t):

(a) If it follows from (21) and restriction 2' (see n. 15) that

the hopeless kind. For example, the propositional part of the method of semantic tableaux (as developed by Beth [2]) could be considered as a systematization of heuristic ideas resulting in a very usable algorithm. The machine programs mentioned above, relying upon this method, are superior also in their propositional parts to the heuristic methods developed and programmed by Newell and Simon; the former being both complete and faster. Beth's [3] assumption that the value of heuristic devices in the field of propositional calculus is insignificant seems justifiable. Another example of a proof algorithm is the proof method proposed in this paper. It seems reasonable to assume that it would be of no advantage to use heuristics in place of this algorithm.

two constants with different cycle numbers are equal, then substitute the constant with less cycle number for the one with higher cycle number in CF_t .

(b) If it follows from (21) and restriction 2 that a dummy is equal to a constant, then substitute the constant for the dummy in CF_t . If the dummy is equal to several constants, choose the one with the least cycle number. (Because of restriction 1, it can not follow from (21) that a dummy is equal to two constants which differ in other respects than the cycle number.)

(c) Substitute c_i for the remaining dummies in CF_t transformed according to (a) and (b).

The result of so transforming CF_t is clearly a tautology and is also equivalent to the disjunction

$$(22) \quad (OF'_1 \vee OF'_2 \vee \dots \vee OF'_t)$$

(cf. phase II), where OF'_i ($i=1, \dots, t$) is like OF_i (see phase I) except for differences in the arguments of the predicate symbols (due to the transformations (a)–(c)). Thus, OF'_i ($i=1, \dots, t$) can be written

$$(23) \quad (\sim F_1^i \vee \dots \vee \sim F_f^i \vee F_{f+1}^i \vee \dots \vee F_g^i)$$

where F_j^i ($j=1, \dots, g$) is like F_j^i in (15) (preparatory step IV) except for differences in the arguments of the predicate symbols. Let Γ_i ($i=1, \dots, t$) be the conjunction $(F_1^i \& F_2^i \& \dots \& F_f^i)$ and let Θ_i be the disjunction $(F_{f+1}^i \vee F_{f+2}^i \vee \dots \vee F_g^i)$. Then, as (22) is a tautology, it follows that so is

$$(25) \quad \Gamma_1, \Gamma_2, \dots, \Gamma_t \rightarrow \Theta_1, \Theta_2, \dots, \Theta_t.$$

We now have to verify that the given sequent $\Gamma \rightarrow \Theta$ is obtainable from (25) by applying the six rules of proof of Gentzen's theorem (i.e. that (25) is a *saf* of $\Gamma \rightarrow \Theta$). (If there is a formula in Γ or Θ in which only C-quantifiers occur, it is necessary to extend the rules PR 3a and b to apply also to E- and U-quantifiers respectively. Below, I will suppose that the rules are so extended.)

Let us speak about a series of appropriate application of the proof rules to a formula in (25) meaning a series of such applications that would result in a formula in Γ or Θ ; an *appropriate*

application of the proof rules is then an application that could belong to a series of appropriate applications to a formula in (25).

Suppose now that no appropriate application of the proof rules can be made to (25) or that there are appropriate applications of the proof rules that applied to the sequent (25) give a sequent S which is different from $\Gamma \rightarrow \Theta$ but to which no appropriate application of the proof rules can be made. I will show that this assumption has the consequence that (21) contradicts the restrictions.

S contains a number of formulas, say G_1, G_2, \dots, G_n , that are different from the formulas in Γ and Θ (otherwise applications of the extended PR 3 a or b would give $\Gamma \rightarrow \Theta$). The reason for there being no more appropriate applications of the proof rules to S must be that such a one would require an application of PR 1 a or b where the constant c , called the *critical constant*, has occurrences, called *critical occurrences*, besides the ones that should be quantified by the application of PR 1 a or b in question (otherwise there would be another appropriate application of the proof rules to S).

Call the critical constant in G_j ($1 \leq j \leq n$) a_j . If there is a critical occurrence of a_j in G_j , then (21) contradicts the restrictions, which is seen as follows. The critical occurrence of a_j must have been substituted for a dummy, d , in G_j at the transformation (b) above, i.e. the identity $a_j = d$ follows from (21) (and restriction 2). This contradicts restriction 3 as $d < a_j$ according to that restriction.

Suppose there is no critical occurrence of a_j in G_j for any j . Then the critical occurrence of a_1 occurs in another formula than G_1 , say G_2 . Again, a_1 must have been substituted for a dummy d in G_2 , and we have $a_1 = d$ (from (21)) and $d < a_2$ (restriction 3), which gives $a_1 < a_2$. If a_2 occurs in G_1 we also get $a_2 < a_1$. Hence, using the same argument n' times ($n' \leq n$) we get $a_1 < a_2 < \dots < a_n < a_j$ where $1 \leq j < n'$.

§ 6.2. The completeness of the proof method

To prove the completeness of the proof method, i.e. that every valid sequent is provable with the method, suppose that the

sequent $\Gamma \rightarrow \Theta$ is valid and that the formulas are in prenex normal form. According to Gentzen's theorem (as stated in § 1.3), there is then a tautological saf of $\Gamma \rightarrow \Theta$. Let S be a tautological saf of $\Gamma \rightarrow \Theta$ and let n be the highest number of associated formulas of some formula in $\Gamma \rightarrow \Theta$ that occur in S . Choose S so that n is as small as possible. Add formulas to S so that the resulting sequent S' contains n associated formulas of every formula in $\Gamma \rightarrow \Theta$ in such a way that S' is still a saf of $\Gamma \rightarrow \Theta$. (In case $\Gamma \rightarrow \Theta$ contains a formula with only C-quantifiers it is necessary to extend PR 3 a and b to apply also to quantifier-free formulas.) Write the sequent S' as one formula in conjunctive normal form and call the result CF'_n . Clearly, every conjunction clause of CF'_n is a tautology.

Let CF_n be the n :th cycle formula obtained by applying the proof method to the sequent $\Gamma \rightarrow \Theta$ (phase II). It is easily verified that if CF_n is written in an appropriate conjunctive normal form CF_n and CF'_n differ from each other only in respect to the arguments of the atomics. Clearly, there is now a function, f , which takes the constants and dummies in CF_n as arguments and the constants in CF'_n as values in such a way that if a constant or dummy, a , has an occurrence o in CF_n , then $f(a)$ has such an occurrence o' in CF'_n that o and o' occupy corresponding places in CF_n and CF'_n .

As CF'_n is a tautology, there is a couple of disjunction clauses such that one is the negation of the other in every conjunction clause M'_i of CF'_n . Choose such a couple from every conjunction clause of CF'_n . Corresponding to such a couple there is an identity list, $IL_{i,j}$, that belongs to the identity clause M_i of CF_n (phase III). It can now be shown that the conjunction of these identity lists

$$(28) \quad (IL_{1,j_1} \ \& \ IL_{2,j_2} \ \& \ \dots \ \& \ IL_{m,j_m})$$

(m is the number of conjunction clauses of CF_n) does not contradict the restrictions listed in phase IV. Then the conjunction of the identity conditions (19) does not either contradict the restrictions, and hence the sequent $\Gamma \rightarrow \Theta$ is provable in cycle n with the proof method (phase IV).

That we can not infer any contradiction from (28) by applying restriction 1-3 in phase IV and the laws of the propositional calculus with equality and inequality, is seen as follows.

(a) If the identity $a=a'$ occurs in (28), then $f(a)=f(a')$ (because of the construction of (28)).

From the restriction on c in PR 1 a and b we get (b) and (c) below.

(b) If a and a' are two constants in CF_n such that $a \neq a'$ according to restriction 1, then $f(a) \neq f(a')$.

(c) If $f(a)=f(a')$ and a and a' are two constants in CF_n like the ones in the premiss of restriction 2, then for all dummies a_1 and a'_1 in CF_n like the ones in the consequence of restriction 2, it holds that $f(a_1)=f(a'_1)$.

Let the constants in CF'_n be c_1, c_2, \dots , and let us adopt the convention that the constant c in PR 1 a or b is the constant with the least index that does not occur in the sequent below the line. If $i < j$, then let us say that $c_i < c_j$. Then we get (d) below.

(d) If a is a dummy $d(p, q, s)$ and a' a constant $c(p, q', s)$ and $q < q'$, then $f(a) < f(a')$.

(e) Any application of the rules of propositional calculus with equality and inequality to equalities and inequalities between dummies and constants in CF_n is of course also possible to equations and inequalities between the corresponding values of the function f .

(a)-(e) give by induction that any inference about identities between constants and dummies a_1, \dots, a_k that can be drawn from (28) with the help of restrictions 1-3 and the propositional calculus with equality and inequality, also holds for $f(a_1), \dots, f(a_k)$. Thus if a contradiction $(a=a' \ \& \ a \neq a')$ or $(a < a' \ \& \ a \not< a')$ were inferable from (28), then $f(a)=f(a')$ and $f(a) \neq f(a')$ or $f(a) < f(a')$ and $f(a) \not< f(a')$. Hence no contradiction can be inferred from (28), which thus does not contradict the restrictions.

I will now in § 7 consider some modifications of the proof method.

§ 7.1. Rules when transforming formulas to prenex normal form

The proof procedure will be shortened if we follow certain rules

when transforming the formulas to prenex normal form (preparatory step I). When using these rules, we have to know whether a quantifier will become a C- or D-quantifier when the formula is finally brought in prenex normal form. We call a U-quantifier that will become a C-quantifier a *CU-quantifier*, and in the same way we speak about *CE-*, *DU-*, and *DE-quantifier*. Before applying the rules, we contract the scope of the quantifiers as much as possible according to the ordinary rules. We then transform to prenex normal form applying the following rules when possible:

Rule 1. When there is an alternative as regards the order in which to write a CU- and a DE- or a CE- and a DU-quantifier, place the CU- or the CE-quantifier to the left of the DE- or the DU-quantifier.²³

Rule 2. Transform

- (a) $(CU_xA \ \& \ CU_yB)$ to $CU_z(A(z/x) \ \& \ B(z/y))$
- (b) $(DU_xA \ \& \ DU_yB)$ to $DU_v \dots DU_w(A(v/x) \ \& \ B(w/x))$
- (c) $(CE_xA \vee CE_yB)$ to $CE_z(A(z/x) \vee B(z/y))$
- (d) $(DE_xA \vee DE_yB)$ to $DE_v \dots DE_w(A(v/x) \vee B(w/y))$.

x, y, z, v, w denote variables, and A and B , formulas. z, v , and w are supposed not to appear free in A or B , and v and w are supposed to be distinct. In place of the dots in rule 2(b) and (d), there may be a number of other quantifiers. It is advantageous in 2(b) and (d) to place C-quantifiers between the D-quantifiers, thus applying rule 1. We may change the place of the two D-quantifiers for this end.

The advantage of rule 1 is easily seen by observing that restriction 3 is less often applicable if we follow this rule. Thus the number of cycles required for proving some sequents is lessened.

The effect of rule 2 is that different constant occurrences more often become occurrences of the same constant (rule 2(a) and 2(c)) and different dummy occurrences more often become occurrences of different dummies (rule 2(b) and 2(d)) (i.e. after the transformations in preparatory step II and III). The following situation will then be less frequent: a_1 and a_3 are constants such

²³ Rule 1 is also suggested in Quine [18].

that $a_1 \neq a_3$ according to restriction 1 in phase IV, a_2 is a dummy, and the identities $a_1 = a_2$ and $a_2 = a_3$ are members of the identity lists formed in phase III. As $a_1 = a_2$ and $a_2 = a_3$ give $a_1 = a_3$, the conjunction of the identity conditions will less often contradict restriction 1 if we apply rule 2.

Sometimes an application of rule 1 makes it impossible to apply rule 2(a) or (c). There seems to be no simple rule for choosing the most advantageous application in these cases.

§ 7.2. *Omitting the transformation to prenex normal form*

The procedure will sometimes be shorter if we do not transform the formulas to prenex normal form. Still, the best strategy is not always to contract the scope of the quantifiers as much as possible. Sometimes it will be more advantageous to apply one of the rules 2(a) and 2(c) in § 7.1. (The problem of choosing between these two alternatives, is the same as that of choosing between an application of rule 1 and 2(a) or 2(c) in § 7.1.) I will briefly outline how the proof method could be modified if preparatory step I was omitted.

A quantifier is then a C- or D-quantifier according as it would be a C- or D-quantifier if the formula was transformed to prenex normal form (cf. definition 1, § 3.1).

We introduce a fourth index of the dummies and the complex constants, called *scope index*. Let the third index, r , in $c(p, q, r, s)$ and $d(p, q, r, s)$ be the scope index and let the other indices be as before (preparatory step III). The scope index of a well-formed part of a formula F is defined as follows:

(a) F has the scope index 1.

(b) If G is a well-formed part of F and has the scope index n , then if G is $\sim H$, UxH , or ExH the scope index of H is also n , and if G is $(H \circ I)$ where \circ is any binary sentence connective the scope index of H is $10 \cdot n$ and the scope index of I is $10 \cdot n + 1$.

The scope index of a quantifier Q in a formula F is the same as the scope index of the scope of Q .

The q index of a dummy or a complex constant previously got by counting the quantifiers in a formula from the left to the right

(preparatory step III), is now got by counting only the quantifiers in the formula that have the same scope index.

Example: In the formula $Ux(EyF \supset (G \vee Uy \sim EzH))$ the scope index of Ux is 1, of Ey , 10, and of Uy and Ez , 111. The q -index of all quantifiers except for Ez is 1. The q -index of Ez is 2.

The scope index is always a sequence of figures 0 and 1, which fact will now be employed when restriction 3 in phase IV is changed for.

Restriction 3'. If $r=r'$ and $q < q'$ or if $r \neq r'$ but the sequence r forms an initial part of the sequence r' , then $d(p, q, r, s) < c(p, q', r', s)$.

§ 7.3. Interval index

It is an advantage if two different dummy occurrences are occurrences of different dummies (as is the effect of rule 2(b) and (d) see § 7.1). A device having this effect will now be described.

We transform the origin formula (preparatory step IV) to conjunctive normal form in two steps as follows. First, we write every formula $\sim F'_i, i \leq f$, and $F'_j, f < j \leq g$, (index as in (15)), in this form. If indexed in an obvious way, we can then write the p :th formula

$$(29) \quad (K_{p,1} \& K_{p,2} \& \dots \& K_{p,m_p})$$

where $K_{p,n}$ ($n=1, \dots, m_p$; m_p is the number of conjunction clauses of the p :th formula or its negation transformed to conjunctive normal form) is a disjunction of atomics and negated atomics. A conjunction clause of the origin formula transformed to conjunctive normal form, called OF^* , can be written as a disjunction of g different $K_{p,n}$, one from each of the g formulas. I.e. a conjunction clause of OF^* has the form

$$(K_{1,n_1} \vee K_{2,n_2} \vee \dots \vee K_{g,n_g})$$

where the sequence n_1, n_2, \dots, n_g is any sequence where $n_p \leq m_p$ ($p=1, \dots, g$; m_p is as in (29)).

There are $\prod_{p=1}^g m_p$ such conjunction clauses of OF^* , and we enumerate them so that their corresponding sequences n_1, n_2, \dots, n_g

come in lexicographic order. Every conjunction clause of OF^* gets thus a number, and with this enumeration in mind we define the k :th interval of the p :th formula as the sequence consisting of i conjunction clauses of OF^* having consecutive numbers, such that the first number is j , $i = \prod_{x=p}^{g-1} m_x$ and $j = 1 + (k-1) \cdot i$.

The characteristic feature of an interval of the p :th formula is that its first member is a conjunction clause of OF^* containing $K_{p,1}$ and that the different $K_{p,n}$ ($n=1, \dots, m_p$; m_p is as in (29)) occur an equal number of times in the interval.

If a dummy $d(p, q, 1)$ in a formula F is such that there is no constants $c(p, q', 1)$ where $q' > q$, we now note that two occurrences of the dummy that are in different intervals of F , can be treated as occurrences of different dummies. We can add an *interval index* to such dummies. The value of the interval index of an occurrence of a dummy in a formula F is set when we form the conjunction clauses of OF^* , so that it becomes equal to the number of the interval of F to which the occurrence of the dummy belongs. Dummies with different interval indices can then be regarded as different when determining whether the conjunction of the identity conditions contradicts the restrictions.

To see that the proof method remains sound if we use the device described in this section, we modify the soundness proof in § 6.1 as follows. OF'_i can now not be written as a disjunction (23). Instead we now make successive transformations of OF'_i starting in the following way. The conjunction of the members of the j :th interval of the g :th formula as they occur in OF'_i (i.e. as they have been transformed according to phase I and (a)–(c) in § 6.1) can be written

$$(I_g^{i,j} \vee K_{g,1}^{i,j}) \& (I_g^{i,j} \vee K_{g,2}^{i,j}) \& \dots \& (I_g^{i,j} \vee K_{g,m_g}^{i,j})$$

where $K_{g,n}^{i,j}$ ($n=1, \dots, m_g$; m_g as in (29)) is like $K_{g,n}$ in (29) except for differences in the arguments of the predicate symbols. OF'_i is the conjunction of these conjunctions when $j=1, \dots, h_g$, where h_g is the number of intervals of the g :th formula (i.e. $h_g = \prod_{x=1}^{g-1} m_x$; m_x is the number of conjunction clauses of the x :th

formula as in (29)). If we apply the distributive law to the conjunction displayed above, we get

$$(I_g^{i,j} \vee (K_{g,1}^{i,j} \& K_{g,2}^{i,j} \& \dots \& K_{g,m}^{i,j}))$$

or $(I_g^{i,j} \vee F_g^{i,j})$, where $F_g^{i,j}$ is like F'_g in (15) except for differences in the arguments of the predicate symbols. OF'_i now becomes.

$$(30) \quad (I_g^{i,1} \vee F_g^{i,1}) \& (I_g^{i,2} \vee F_g^{i,2}) \& \dots \& (I_g^{i,h_g} \vee F_g^{i,h_g})$$

(30) is transformed by appropriately existentializing ("appropriately" is used in the same way as the notion of an appropriate application of the proof rules in § 6.1) all constants in every formula $F_g^{i,j}$ ($j=1, \dots, h_g$) that have been substituted at the transformations (b) and (c) in § 6.1 for dummies with interval index. The quantified $F_g^{i,j}$ then become like F_g (as obtained after preparatory step I) in respect to the D-quantifiers that are not succeeded by any C-quantifier and the variables quantified by these quantifiers. All the quantified $F_g^{i,1}, F_g^{i,2}, \dots, F_g^{i,h_g}$ then become identical; call such a formula F_g^i . (30) can then be written

$$((I_g^{i,1} \& I_g^{i,2} \& \dots \& I_g^{i,h_g}) \vee F_g^i).$$

We continue these transformations in a similar manner with the formulas $(g-1), (g-2), \dots, (f+1), f, (f-1), \dots, 1$ in place of the formula g , observing that the last f formulas are to be generalized instead of existentialized as they are negated. The result of so transforming OF'_i can be written

$$(31) \quad (\sim F_1^i \vee \dots \vee \sim F_f^i \vee F_{f+1}^i \vee \dots \vee F_g^i).$$

The disjunction of all disjunctions (31) ($i=1, \dots, t$) is of course valid. We then continue the argument as in § 6.1 with F_p^i in place of F_p^i ($i=1, \dots, t; p=1, \dots, g$) except that the sequent (25) now obtained is not a saf of $I' \rightarrow \Theta$ as the formulas of the sequent (25) now obtained contain all the D-quantifiers that are not succeeded by a C-quantifier in the sequent $I' \rightarrow \Theta$. Clearly, the validity of the sequent (25) now obtained also implies the validity of $I' \rightarrow \Theta$.

§ 7.4. *C-quantifiers not succeeded by any D-quantifier*

To simplify the problem in phase IV of determining whether the conjunction of the identity conditions contradicts the restrictions, we can strengthen restriction 2 by adding: If there is no dummy $d(p, q', 1)$ where $q' > q$, then $c(p, q, s) \neq c(p, q, s')$.

By making this addition we spare ourselves from considering some unpromising values of the dummies without affecting the question whether the conjunction of the identity conditions contradicts the restrictions. That this is so, is evident from the fact that if there is a valid saf of $\Gamma \rightarrow \Theta$, then there is such a saf with the same number of formulas that is obtained by so applying the transformation rules that the same constant is never substituted at two applications of TR 2 a or b to the same formula. Since TR 3 a or b is applied only to formulas beginning with a D-quantifier, the following is then true: If there is no dummy $d(p, q', 1)$ where $q' > q$ and if there is a valid saf of $\Gamma \rightarrow \Theta$, then there is such a saf with the same number of formulas where $f(c(p, q, s)) \neq f(c(p, q, s'))$ always holds (f is the same function as in § 6.2).

Hence, if no quantifier prefix of the formulas in Γ and Θ brought into prenex normal form is more complex than $Cx_1Cx_2 \dots Cx_kDy_1Dy_2 \dots Dy_mDz_1Cz_2 \dots Cz_n$ (as is the case in Skolem normal form), then restriction 2 can be omitted and restriction 1 strengthen to exclude any identity between two constants.

§ 7.5. *A proof method without inequality*

We can modify restriction 3 so that we only have to deal with equalities and not with inequalities when determining the question in phase IV whether the conjunction of the identity conditions contradicts the restrictions. The effect of this is to fix the relative order between the applications of the transformation rules, which will often give longer derivations than before, i.e. we will often need more cycles than before to prove a sequent.

First we define the *degree* of a quantifier in the formulas resulting from step II. All D-quantifiers in a formula not succeeded by a C-quantifier have the degree 1, and all C-quantifiers in a

formula not succeeded by a D-quantifier have the degree 2. All C-quantifiers succeeded by D-quantifiers of degree r but of no higher degree have the degree $r+1$, and all D-quantifiers succeeded by C-quantifiers of degree r but of no higher degree have the degree $r+1$.

To all dummies and complex constants we then add a fourth index, a degree index, denoting the degree of the corresponding quantifier. Let the third index in $c(p, q, r, s)$ and $d(p, q, r, s)$ be the degree index. We then change restriction 3 for

Restriction 3''. If $c(p, q, r, s) = d(p', q', r', s')$ and either $s = s'$ and $r < r'$ or $s > s'$, then

if $r < r'$ there is some $x < s'$ such that $c(p, q, r, s) = c(p, q, r, x)$,
and if $r > r'$ there is some $x \leq s'$ such that $c(p, q, r, s) = c(p, q, r, x)$.

Thus, if $s = s' = 1$ and $r < r'$, then $c(p, q, r, s) \neq c(p, q, r, s')$.

That the proof method remains sound after these modifications is seen as follows. Suppose the sequent $\Gamma \rightarrow \Theta$ is provable with the modified proof method. Then there is a conjunction (21) (see § 6.1) of identity lists that does not contradict restrictions 1, 2, 3''. Transform (21) in the following way:

If it follows from (21) and restrictions 2 and 3'' that there is an x , such that $x < s'$ and $c(p, q, r, s) = c(p, q, r, x)$, then choose an $s'' < s'$ such that the conjunction of (21) and $c(p, q, r, s) = c(p, q, r, s'')$ does not contradict the restrictions (as (21) does not contradict the restrictions there exists such an s''). Transform (21) by forming the conjunction of (21) and the identity $c(p, q, r, s) = c(p, q, r, s'')$.

We can now order the constants and dummies as follows.

(i) If it follows from the transformed (21) and restriction 2 that some complex constants a_1, a_2, \dots, a_n , are equal, then they are all to have the same order as the one with the lowest cycle index (i.e. the fourth index).

The constants not ordered by (i) are ordered by (ii)–(iv):

(ii) of two complex constants with different cycle numbers the one comes first whose cycle number is lowest

(iii) of two complex constants with the same cycle number the one comes first whose degree is highest

(iv) all simple constants come first in the enumeration.

(v) If it follows from the transformed (21) and restriction 2 that a dummy is equal to a constant, the dummy is to have the same order as the constant. The other dummies are to have the same order as c_1 .

All the constants and dummies are now ordered in such a way that if it follows from (21) restriction 2 and 3 that $a > a'$, then a comes before a' in the order. Thus, (21) does not contradict the restrictions 1, 2, and 3.

To see that the proof method remains complete, we have to make use of another version of Herbrand's theorem than the one stated in § 1.1. The new version is called *Herbrand's theorem 2* (abbreviated: *HT 2*):

If A is in prenex normal form, then the sequent $\rightarrow A$ is valid if and only if there exists a valid saf of A

$$(32) \quad \rightarrow A_1, A_2, \dots, A_t$$

fulfilling the following conditions concerning the substitution of a constant c_k ($k=1, 2, \dots$) for a variable x quantified by a universal quantifier Q :

(i) c_k is different from all constants substituted for variables quantified by other universal quantifiers than Q , and also different from the constants that occur in A (if any);

(ii) if c_k is substituted for the variable x to obtain both A_i and A_j , then every variable quantified by a quantifier to the left of Q in A is replaced by the same constant in A_i and A_j ;

(iii) if c_k is substituted for x to obtain A_i but not substituted for x to obtain any A_j where $j < i$, then for every c_m that occurs in the formula A_i as substituted for a variable quantified by a quantifier to the left of Q in A , or that occurs in a formula A_j where $j < i$, it holds that $k > m$.²⁴

The completeness proof in § 6.2 can now be used with HT 2 instead of Gentzen's theorem and some other modifications. The

²⁴ For proof that the validity of $\rightarrow A$ implies the validity of a sequent (32), we can use Dreben [6], who states that if A is valid there exists a certain Herbrand Tautology. It is easily verified that a Herbrand Tautology satisfies the conditions of (32). (For proof of his statement Dreben refers to Gödel.)

construction of CF'_n is now a little different. Let $F_1, \dots, F_f \rightarrow F_{f+1}, \dots, F_g$ be the valid sequent $\Gamma \rightarrow \Theta$. The sequent

$$\rightarrow (\sim F_1 \vee \dots \vee \sim F_f \vee F_{f+1} \vee \dots \vee F_g)$$

consisting of only one formula, is then also valid. Rewrite this formula in prenex normal form in such a way that the quantifiers of greater degree come before the ones of less degree. Call the obtained formula A . According to HT 2 there is then a valid satisfiability of A . Let the sequent $\rightarrow A_1, A_2, \dots, A_n$ be one with the least possible n , and use the result of writing $(A_1 \vee A_2 \vee \dots \vee A_n)$ in conjunctive normal form as CF'_n .

(b) and (c) in § 6.2 now follow from the conditions (i) and (ii), respectively, in HT 2. Finally, (d) is changed for

(d') If $f(a) = f(a')$ and a is a constant in CF_n and a' is a dummy in CF_n both like the ones in the premiss of restriction 3'', then it follows from condition (iii) in HT 2 that there exists a constant a_i in CF_n such that $f(a) = f(a_i)$ and a_i is like a except for the cycle number, which is less than that of a' if the degree of a is less than that of a' and which is equal or less than the cycle number of a' if the degree of a is greater than that of a' .

§ 8. Not provable formulas

In this section, I will show how the proof method can sometimes be used to prove that a given sequent $\Gamma \rightarrow \Theta$ does not hold, i.e. as a refutation method. (A very simple case is already included in (a) of phase III.) The case to be considered is essentially the special case of the decision problem solved by Herbrand for a formula in prenex normal form whose matrix is a disjunction of atomics and negated atomics (see e.g. Ackerman [1], p. 85, Church [5], p. 256, or Hilbert and Bernays [9], pp. 160 and 161). As will be shown below, if a formula is as just described, then it is either provable with our proof method in cycle 2 or it is not provable at all. The case to be treated below is an obvious extension obtained by the following observation: Let QM be a formula in prenex normal form where Q is the quantifier prefix and M is the quantifier-free matrix, and let M be in conjunctive

normal form $(M_1 \& M_2 \& \dots \& M_m)$ where M_i ($i=1, \dots, m$) is a disjunction of atomics and negated atomics. Then, QM is not provable if QM_i is not provable, and the latter formula is in the form required in Herbrand's case of the decision problem.

Let us first define an enumeration of the conjunction clauses of a cycle formula CF_s supposing that the origin formula, OF, is written in conjunctive normal form. If s is 1, the i :th conjunction clause, M_i , of CF_s is the same as the i :th conjunction clause of OF_1 (for an enumeration of OF we can use the one defined in § 7.3). If $s > 1$, M_i of CF_s is the disjunction of the j :th conjunction clause of CF_{s-1} and the k :th conjunction clause of OF_s , where $n \cdot (j-1) + k = i$ and n is the number of conjunction clauses of OF. (Thus, M_1 is the disjunction of the first conjunction clause of CF_{s-1} and the first conjunction clause of OF_s ; M_2 , the disjunction of the first and second conjunction clause of CF_{s-1} and of OF_s respectively; M_{n+1} , the disjunction of the second and first conjunction clause of CF_{s-1} and OF_s respectively, etc.)

Let us now introduce the notion of *simple conjunction clause of type i* as follows. A conjunction clause M_i of CF_1 is a simple conjunction clause of type i . If M_i of CF_s is a simple conjunction clause of type j , then M_k of CF_{s+1} is a simple conjunction clause of type j if $k = (i-1) \cdot n + j$ where n is the number of conjunction clauses of OF transformed to conjunctive normal form. In other words, a simple conjunction clause of type i of CF_s , $s > 1$, is a disjunction

$$(M_i \vee M^a \vee M^a \vee \dots \vee M^s)$$

where M_i is a conjunction clause of CF_1 and M^j ($j=2, \dots, s$) can be obtained from M_i by just substituting j for the cycle index (i.e. the last index) of the dummies and complex constants. (A conjunction clause that is not simple can not be obtained from just one conjunction clause of CF_1 in this way.)

The following now holds:

If there is a simple conjunction clause M_i of CF_2 , whose associated identity condition IC_i contradicts the restrictions in phase IV, then the given sequent $\Gamma \rightarrow \Theta$ does not hold.

Proof. Suppose that there is a simple conjunction clause M_i of

type t of CF_2 such that IC_i contradicts the restrictions. Then, every identity list $IL_{i,j}$ ($j=1, \dots, n_i$; n_i as in (18)) contradicts the restrictions. $IL_{i,j}$ is a conjunction

$$(32) \quad (a_1=a'_1 \ \& \ a_2=a'_2 \ \& \ \dots \ \& \ a_k=a'_k)$$

(cf. (17)), where the cycle indices of all the a_h ($h=1, \dots, k$) are identical and similarly for all the a'_h .

We now consider another simple conjunction clause, $M_{i'}$, of type t of some arbitrary cycle formula CF_s . Then we notice that every identity list $IL_{i',j}$ of $IC_{i'}$ in cycle s can be obtained from some identity list (like (32)) of IC_i in cycle 2 where the cycle numbers of a_h and a'_h ($h=1, \dots, k$) are different (i.e. one is 1 and the other is 2) by substituting and appropriate number s' for the cycle index of every a_h in (32) and an appropriate number s'' for the cycle index of every a'_h in (32) ($h=1, \dots, k$).

But from the identity list (32) of IC_i in cycle 2, we can infer a contradiction by applying the restrictions 1-3 and rules for the propositional calculus, equality and inequality. It is then easily verified that we can infer a similar contradiction from an identity list that is obtained from (32) by the substitutions mentioned. Indeed, suppose $IL_{i',j'}$ (in cycle s) is obtainable from $IL_{i,j}$ (in cycle 2) by the substitution of s' for the cycle index 1 and s'' for the cycle index 2 in the dummies and complex constants in $IL_{i,j}$. Make the same substitutions in all the dummies and complex constants in the inference (of the kind described above) from $IL_{i,j}$ which ends with a contradiction. We then obtain an inference of the same kind from $IL_{i',j'}$ that ends with the contradiction. Hence all the identity lists of $IC_{i'}$ in cycle s contradict the restrictions, and according to phase IV, the given sequent $\Gamma \rightarrow \Theta$ is then not provable in cycle s .

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DISCUSSIONS

Berkeley and Mental Acts. By Harry M. Bracken. (State University of Iowa.)

In a recent article¹ in this journal on Berkeley's doctrine of notions, Grossmann has argued that Berkeley's failure to spell out the doctrine may well stem from an awareness of a peculiar philosophical difficulty. Grossmann maintains the thesis that for Berkeley, to be known notionally is the same as to be known through an act. There can, however, be no acts of sensing because that would make our acquaintance with *sensa* what it is not – i.e. notional. Furthermore, he argues that the classical view of acts is that they are relational, but that Berkeley was bound to reject such acts because he was committed to the view that descriptive relations do not exist. Accordingly, when Berkeley set about discussing notions, he ran into the paradox of having to explain how various existents could be "connected" by an act which, in view of the denial of relations, could not be both an existent and relational. Grossmann concludes that Berkeley's reluctance to deal with notions may very well stem his suspicion that the problems of the non-relational act are wellnigh insurmountable.

I cannot deny that one may wish that Berkeley had dealt with notions at greater length, but I do propose to argue that there are good historical reasons for thinking that whatever may have inhibited the development of Berkeley's theory, it was not the non-(descriptive) relational act. Malebranche, for example, from whom Berkeley learned a good deal of philosophy, appears to have had one. That is no guarantee that the concept is intelligible, but it at least offers us an opportunity to explore its role.

First, however, it should be recalled that insofar as Berkeley denies acts of *sensing*, he is standing in a long tradition. From Plato to Moore,² philosophers have been hesitant to consider *sensa*, taken as the objects of such acts, as plausible objects of knowledge. Perhaps the tersest statement of the conflict between 'sensing' and 'knowing' is to be found in

¹ Reinhardt Grossmann, "Digby and Berkeley on Notions".

² I have in mind his remarks in "A Reply to my Critics", sect. 10, pp. 653 ff., in *The Philosophy of G. E. Moore*, ed. P. A. Schilpp (Evanston and Chicago: 1942).

Prichard's essay, "The Sense Datum Fallacy."³ And of course the core of the difficulty, whether in the *Theaetetus* or Berkeley, in Moore or Prichard, arises – rightly or wrongly – from their suspicions and/or convictions that *sensa* are (mind) dependent entities. Toothaches, unlike numbers, are not 'real' in the way that knowledge requires. For his part, Berkeley proceeded to couple a denial of acts of sensing with the affirmation of *esse* is *percipi* and thus to attack a certain sort of representative realism. At least partially as a result of this, he subsequently became concerned with the problem of the proper object of knowledge, as I shall discuss below.

Malebranche⁴ was also troubled with the problem of the proper object of knowledge. He postulated Intelligible Extension as composing the realm of objects of knowledge. These are the Ideas, resident in God, which we may come face to face with. To know the Ideas is to know the natures of things. These Ideas were taken by Malebranche to be the entities which *represent* all things to us (and hence were called *êtres représentatifs* by his critic, Arnauld). They represent things in the sense that geometry makes extended things intelligible. Granted the Cartesian identity of the essence of material things with extension, it follows that *if* any material thing exists, its essence is comprehended in the theorems of Euclid. Whether, of course, there *are* any things – is not a theorem; it is not an element in our knowledge (except as a result of acceptance of the Biblical revelation that God did will the existence of things). The effect of this is to make corporeal things virtually dispensable. Furthermore, the Ideas were to be distinguished sharply from our perceptions or subjective ideas. These latter sorts of things Malebranche argued, are merely modifications of a spiritual substance, and their *esse* is presumably *percipi*. They simply do not have the ontological character appropriate to epistemic objects, and they do *not* serve as stepping stones to such objects by way of abstraction. Malebranche, then, has (a) spirits, (b) modifications of spirits, e.g. perceptions and feelings, (c) Ideas, (d) Material Things.

Arnauld challenged Malebranche almost immediately. He argued that (c) was superfluous: Malebranche had simply failed to see that his own perceptions, his own mental acts, were *already* representative. He maintained further that (b), at least in the case of perception, could be viewed from two different aspects:

I have said that I take the *perception* and the *idea* to be the same thing. It is nevertheless necessary to note that this thing, although

³ Reprinted in H. A. Prichard, *Knowledge and Perception* (Oxford: 1950).

⁴ Cf. esp. *De la Recherche de la Vérité*, Bk. III, Pt. 2 and X^e *Éclaircissement*. Also H. Gouhier, *La Philosophie de Malebranche et son expérience religieuse*, 2nd. ed. (Paris: 1948) esp. p. 214.

unique, has two relations: the one to the soul that it modifies, the other to the thing perceived, insofar as it is objectively in the soul, and that the word *perception* denotes more directly the first relation, and that of *idea* the latter. Thus the *perception* of a square denotes more directly my soul as perceiving a square; and the *idea* of a square denotes more directly the square, insofar as it is *objectively* in my spirit.⁵

We can thus distinguish in perception (1) the spiritual substance, (2) the act of perceiving, (3) the conceptual content *by* which we know, and (4) the material thing known. The relation that (2) establishes between (1) and (3) is presumably logical. But it seems unlikely that anyone, operating within a substance/attribute context, would wish to call the relation that (2)-(3) establishes between (1) and (4) 'descriptive'. The relation between (3) and (4) is a most peculiar one and I think reflects the difficulty in accurately characterizing the relational aspect of mental acts. Arnauld's *ideas* function a good deal like St. Thomas' intelligible species,⁶ i.e. they are the objects known "existing immaterially" in the mind, and also, these conceptual entities are *not* that which we know but that *by* which we know. Admittedly the relation between (3) and (4) is one of 'similitude' between spiritual and material existents. But I think it is clear that historically 'similitude' has proved difficult to explicate – whether in St. Thomas or the Cartesians. In any case, as far as Arnauld could see, his own *ideas* did everything necessary to account for our perception of material things. Thus either in regard to (2) or (3) or both in combination, there is no reason to think that he subscribed to the doctrine of the (descriptive) relational act, and every reason to think he did not.

In response, Malebranche argued⁷ both that we are united with Intelligible Extension, and also that no merely mental *idea* could represent anything. The trouble with Arnauld's *ideas* was that they were (in our sense) subjective. It was inconceivable to Malebranche that such

⁵ Arnauld, *Des vraies et des fausses idées* in *Oeuvres Philosophiques de Antoine Arnauld*, ed. Jules Simon (Paris: 1843), p. 51 (ch. V). See also the *Réponse de Malebranche*, pp. 267 ff., Cf. ch. X.

⁶ Arnauld comments favorably on St. Thomas' doctrine, see *Des vraies*... Ch. XIII. G. A. Johnston suggested John Sergeant as a likely source for Berkeley's doctrine of notions, whereas Grossmann argues for the man Sergeant admired: Sir Kenelm Digby. John W. Yolton has noted the strong resemblances between Sergeant's account of notions and the discussion in St. Thomas' *Summa Theol.* Q. 84, art. 1; "Locke's Unpublished Marginal Replies to John Sergeant", *Journal of the History of Ideas*, XII (1951), 528-559; esp. 551-2.

⁷ See his *Réponse*, ch. V ff., also parts of *Recherche* cited above.

'private' entities could comprehend eternal and immutable Ideas. If one knows something, one must be united with it – Malebranche's formulation of a traditional doctrine. But also for traditional reasons, the thing-known must exist independently of the knower. Finally, it is simply the case that our spiritual modifications are merely felt. We do not know their natures nor how they are related either to other modifications or to spirits. To *know* spirits would require an Idea. It is absurd to maintain (as Arnauld did) that such vague and uncharted perceptions, themselves not truly intelligible, could serve to make radically different (i.e. material) substances – of which they clearly are *not* modifications – known to us.

Thus we do not 'see' the immutable Ideas via sentiments – we 'see' them directly – and they are things wholly different from our feelings or sentiments. There is some point to using blackboards in teaching geometry, but the mere feelings (sentiments) that occur as we look at the board do *not* stand in any known or knowable relation to the Ideas we may also be contemplating. This is one side of Malebranche's occasionalism; the mystery behind the relation of the Ideas to spirits. The other side is the relation of the Ideas to the Material Things. Both are encompassed by the wondrous, but unknown, laws of conjunction of soul and body.

Malebranche's language, whether he is talking of spiritual modifications – perceptions, sentiments, etc., or Intelligible Extension, is always the language of immediate awareness. There is no question that Malebranche has mental acts; the question is rather to determine their nature. Recall that we do not have Ideas of selves, for that would mean that we would have the same sort of (essential) knowledge about spirits and their modifications as we do about material things – it would be 'spiritual geometry'. Recall also that perceptions, sentiments, etc., are radically different from Ideas. The one are purely subjective, the other objective (in our sense). There is no cognisable relation between Spirits and Material Things; as for the spiritual acts which do occur, the denial of their representative function, based as it is in part on our lack of Spiritual Ideas, precludes the very possibility of specifying the nature of the act – beyond the bare assertion of its existence. It is hard to see how the non-(descriptive) relational act could be more baldly affirmed.

Grossmann, it will be remembered, has taken the view that Berkeley's inadequate account of notions arises from the difficulty of holding that notional knowledge involves acts, and at the same time denying the existence of descriptive relations; thus the difficulty with notions is that of the non-relational act. I have sought to indicate that however puzzling acts which are not descriptive relations may be, such acts were nevertheless ingredients in the philosophy of Malebranche (and Arnauld

too) whose influence on Berkeley has long been well established. Given then, that the non-(descriptive) relational act was virtually a characteristic of occasionalism, there seems little reason to conclude that Berkeley should have interpreted its occurrence in his own system as a philosophical dead-end.

However much I may disagree with Grossmann on that point, there is also an important area of agreement. For part of his concern is to establish the thesis that whatever is known notionally, is known through acts. As is also clear from his discussion of notions in Digby, Grossmann sees the problem of notions in relation to the more general question of the nature of knowledge.

There is, of course, a close tie between the problem of acts and that of the object of knowledge. Although it is a further issue whether or not notions can be assimilated to descriptive relational acts, I think it is clear that Berkeley wants as a philosophical consequence of his doctrine of notions what many (philosophical) proponents of acts have sought to have them achieve.⁸ Accordingly, recalling my earlier comments on the topic of whether Berkeleian ideas of sense are ontologically proper objects of knowledge, let me now add that it is no accident that a key feature in several of the classical refutations of idealism has been the assertion of some sort of mental acts. Realism was a major consideration, at least in the sense that these philosophers sought to guarantee the 'independence'⁹ of the object known, from the knowing mind; i.e. they sought to meet the Platonic requirement that knowledge be of the real. Inasmuch, then, as acts have frequently been introduced with an 'independence' consideration in view, it is hardly surprising that Berkeley should utilize acts or something similar when he himself faced up to the issue.

The failure to appreciate the role of notions vis-à-vis the object of knowledge has led people to apply the abstract-idea argument to notions and spirits. This step, often made in ignorance of the philosophical history of notions, acquired some respectability from the assumption that Hume had originally posed it as an objection. In the light of Popkin's "Did Hume Ever Read Berkeley?"¹⁰ this now seems most dubious.

My own view is that Berkeley envisioned his sense-ideas to be rather

⁸ For a discussion of acts and some related problems, see Gustav Bergmann, "Intentionality", *Archivio di Filosofia*, 1955, reprinted in *Meaning and Existence* (Madison: Univ. of Wisconsin Press, 1959).

⁹ 'Independence' is of course a problematic term. In this connection, see Edwin B. Allaire, "Existence, Independence, and Universals", forthcoming *Philosophical Review*.

¹⁰ Richard H. Popkin, "Did Hume Ever Read Berkeley?" *Journal of Philosophy*, LVI (1959), 535 ff.

like Malebranche's 'sentiments', and that he was able to use this interpretation to attack the Lockean philosophy in which ideas, sensations, concepts, thought, and matter, were hopelessly muddled. This enabled him not only to use it (via *esse is percipi*) as an attack on those forms of scepticism¹¹ which arise from a certain sort of representationism, but it could also vindicate a substantial¹² (and potentially immortal) Self – given the dependent character of ideas entailed by *esse is percipi*. But it failed to do justice to knowledge; for knowledge seemed to call for something real. Hence the appearance of a profoundly different kind of entity: a notion.

I think the best way to appreciate Berkeley's continued involvement with the problem of the object of knowledge is to examine his Divine Language metaphor. In the *New Theory of Vision*, Berkeley took sense-ideas as signs which signified an independent (touchable) world. This 'vulgar error' was corrected in the *Principles*,¹³ but it left signs merely signifying other signs. Berkeley's solution, I have argued elsewhere,¹⁴ was to take his metaphor more seriously: we are to think of colors, smells, tastes, etc., as the letters of His language. To be more precise, the realm of sense-ideas is the realm of marks or signs; the realm of their *meanings*, is the realm of notions. The radical disparity between signs and meanings, is something of an analogue to that between Malebranche's sentiments and his Ideas, and hence one has a sort of 'linguistic Malebranchianism'.

The metaphor also reveals the real absurdity in Locke's formulation of abstraction: abstraction from ideas can no more yield meanings than abstraction from sounds or marks can. Where Malebranche relies on the wondrous but unknown laws of conjunction of soul and body to account for the relation between Ideas and sentiments, Berkeley is equally occasionalistic in appealing to the 'linguistic' learning process to establish the link. Furthermore, Meanings or Notions may function as objects, much as Malebranche's Ideas, yet they have a signal advantage over them. Intelligible Extension raised the spectre of an extended deity (Spinozism) which did not arise from the more intuitively obvious metaphor of the Divine Language. 'Strictly', you and I do not have the same sense-ideas; but then 'strictly', sense knows nothing.¹⁵ To *know*,

¹¹ Cf. Popkin, "Berkeley and Pyrrhonism", *Review of Metaphysics*, V (1951), 223–246.

¹² See, e.g. *Principles* §§ 25–7, in *The Works of George Berkeley*, ed. A. A. Luce and T. E. Jessop (London: 1948–57), 9 vols. See Vol. II, pp. 51–2.

¹³ See *Principles* §§ 43, 44, 65.

¹⁴ See my "Berkeley on the Immortality of the Soul", *Modern Schoolman* (Jan. 1960).

¹⁵ Cf. third of *Three Dialogues*, *Works* II, 245–7; *Siris* § 253, *Works* V, 120.

is to know what He means by His language of sounds, colors, and smells; i.e. we know the rules of His language, His grammar. Regardless of how 'intimately' we may be united with Meanings, they are, like Malebranche's Ideas, 'distinct' from us. Unlike Malebranche's Ideas, they carry fewer subjectivistic connotations; especially when formulated in terms of 'rules'.

Finally, it is worth reminding oneself that Notion, Meaning, and Concept were held to be synonymous by many of Berkeley's predecessors. Berkeley's preference for the first two stems from his sensitivity to the confusions with sensation that had come to obscure the latter, especially as a result of controversies¹⁰ over their purported 'representative' function (in which, as we have seen, Malebranche himself was involved). Thus once it is appreciated that Berkeley's concern with notions is a traditional one – namely, that of the nature of the object of knowledge (which I think Grossmann's article helps us to do), and that Berkeley recognized this problem as early as the *New Theory of Vision*, the temptation to treat notions as a vague afterthought should be removed. Similarly, as Luce and Jessop have long maintained – notions constitute no threat to the 'unity' of the Berkeleian philosophy. No longer mesmerized by the word 'notion', we shall be able to explore the role that this concept plays. Then, and only then, will we be able properly to evaluate Berkeley's treatment of it – for we shall be seeing it in its proper context: a major segment of the Platonic tradition that runs from Malebranche to the Greeks via Sts. Thomas and Augustine.

On a type of "ambiguity". By Robert J. Richman.
University of Oregon.

An extreme case of ambiguity is what is called the *antithetical sense of primal words*. There is evidence that in the oldest language opposites such as: strong-weak, light dark, large-small were expressed by the same root word. Thus, in ancient Egyptian *kev* stood for both strong and weak. In Latin *altus* means high and deep, *sacer* both sacred and accursed . . .¹

Waismann's point is a familiar one. Freud, among others, has alluded to it. He considers the antithetical sense of primal words important with regard to the interpretation of dreams.

¹⁰ Cf. Popkin, "L'abbé Foucher et le problème des qualités premières", *XVII^e Siècle*, No. 33 (1957), 633 ff.

¹ Waismann, F., "Language Strata", reprinted in *Logic and Language, Second Series*, (Oxford, 1955), p. 11.

The governing laws of logic [Freud writes] have no sway in the unconscious; it might be called the Kingdom of the Illogical . . . contraries are not kept apart from each other, but are treated as though they were identical, so that in the manifest dream any element may also stand for its contrary. Certain philologists have found that the same holds good in the oldest languages, and that contraries such as "strong-weak", "light-dark", "high-deep" were originally expressed by the same roots . . . Remains of this original double meaning seem to have survived even in such a highly developed language as Latin in the use of words like *altus* ("high" and "deep") and *sacer* ("holy" and "accursed").²

I should like to raise a question about this type of ambiguity, not from the point of view of philology or of psychoanalysis, but from the point of view of semantical or philosophical analysis. The question may be phrased: should the "antithetical sense of primal words" be accounted a special type of ambiguity; or, rather, *should it be considered as a type of ambiguity at all?* The question so put may sound self-defeating, since 'antithetical sense' clearly seems to imply 'ambiguity'. To avoid foreshortening our discussion, let us take 'antithetical sense' simply as a *label* for the kind of term to which Waismann and Freud refer.

At first blush, nothing would seem more obvious than that terms with antithetical senses are ambiguous. But, as I have attempted to show elsewhere,³ the determination of ambiguity is by no means so straight-forward as we have sometimes believed. To say of a term that it is ambiguous is to say that it connotes at least two distinct properties, or that it refers to two or more distinct classes. A difficulty arises, however, in trying to set forth criteria for determining whether or not in a given case we are dealing with distinct concepts (or with distinct classes). Things can be sorted in many different ways, each sorting being based on different properties or principles of classification and each yielding different classes. Unless we think of properties as "subsistent entities" or of classes as "natural" we cannot say of a given term that it is ambiguous *simpliciter*.

With regard to terms generally it should be clear, first of all, that the fact that a term of one language is translated into more than a single term of another language is of itself insufficient to show that

² *An Outline of Psychoanalysis*, (New York, 1949), p. 53.

³ "Ambiguity and Intuition", *Mind*, January, 1959, I should make clear incidentally, that I am using the term 'having more than a single meaning' without the suggestion of misleadingness which the use of this term frequently conveys. I have discussed this point in the article cited.

the term must be considered ambiguous in the original language. Thus, we are not forced to the conclusion that the English word 'snow' is ambiguous because in Eskimo it is translated into many different words.

To an Eskimo this all-inclusive word ['snow'] would be almost unthinkable; he would say that falling snow, slushy snow, and so on, are sensuously and operationally different, different things to contend with; he uses different words for them and for other kinds of snow.⁴

English-speaking people tend to note *similarities* among these various types of snow, to subsume them under a single concept. This is doubtless due in large measure to the fact that in our language there is a single word for snow. (Which came first, the concept or the word?) But whatever the reason, since these obviously different sub-classes of snow are incorporated into a single class, the English word 'snow' is not ambiguous, despite the fact that in Eskimo the corresponding terms are many.

To take a different example, the verb 'to eat' is not accounted ambiguous because it corresponds to the German 'fressen' and 'essen', even though a German-speaking individual might feel that it should be so accounted. Taking a cue from his language, a German might feel that the eating activities of humans and of other animals are so dissimilar as to represent a difference of kind, and might believe therefore that the English term 'to eat' is ambiguous. Again, however, the speaker of English would tend to subsume eating activities under a single category. If pressed, he might argue that there is no sharp line to be drawn between eating as engaged in by an animal and by a human being. The eating habits of a healthy two-year old, after all, do not resemble those of the Queen of England nearly so much as they do those of the non-human inhabitants of the local zoo.

At any rate, it would seem reasonable to suppose that, as ambiguity is usually conceived of, the judgment as to whether or not a term of a given language is ambiguous cannot be made simply on the basis of the fact that the term is translated into more than one term of some other language. Thus, the fact that the Latin 'altus' can be translated into the English 'high' and 'deep' is not sufficient to show that it is ambiguous.

I should make it clear that I am not trying to argue that, say 'altus', is *really* unambiguous. (After all, whether or not 'altus' is ambiguous is of itself hardly a philosopher's question.) Rather, I am simply trying to show that the question of its ambiguity does not admit of the simple

⁴ Whorf, Benjamin, "Science and Linguistics", in *Collected Papers on Metalinguistics* (Washington, D.C., 1952), p. 6.

answer which Waismann and Freud suggest. I should also like to indicate – but only incidentally – that some of the air of mystery attaching to talk about the “Kingdom of the Illogical” is unnecessary.

How might one argue that terms with antithetical senses are unambiguous? Well, the “argument” would consist largely of getting others to *think* of these terms as unambiguous, to look at them from a new perspective. When we have “loosened up” our thinking to the point of seeing that alternative possibilities of interpretation exist, then we shall be less inclined to speak of a “natural” or of a “right” point of view. (It seems obvious that many philosophical disagreements – or should I say “disagreements”? – arise because of the failure of disputants to see the possibility of different ways of regarding a given question or assertion.)

Let us take as an example the Latin term ‘*altus*’. (For our purposes we may disregard differences in form based on gender, case, and number.) Similar considerations will apply to other terms with antithetical senses. ‘*Altus*’ means high and deep. Therefore it is ambiguous. But let us try to think of ‘*altus*’ as univocal. We may then ask: what do things denoted by ‘*altus*’ have in common? Well, they are all either high or deep. ‘*Altus*’ connotes the (single) complex property of being high or deep. This goes too far – since the same line of reasoning could be used to show that *no* term is ambiguous – but it is a useful beginning. (Many terms which we think of as simple can be conceived of as connoting complex properties. To take the sort of example of a simple term which comes first to mind, ‘red’ can be thought of as connoting the complex property of being scarlet or being crimson or . . .) But much more can be said for the univocality of ‘*altus*’. To apply the term to anything is to make reference to one of the thing’s vertical dimensions. It is to say, in effect, that the thing referred to is extreme or abnormal in this dimension. Bearing this in mind, is it then so clear that we are making an ambiguous assertion when we characterize something by using the term ‘*altus*’? Consider: Must our statements be ambiguous when we speak of political extremists (since such individuals may be of the extreme right or of the extreme left) or when we speak of persons of abnormal intelligence (since the intelligence of such persons may be extremely high or extremely low) or when we speak of logically determinate statements (since such statements may be logically true or logically false)? Rather we might note that there are terms in English (as well as in other natural languages) which serve to pick out individuals at opposite ends of some spectrum. Such terms might be conceived of as *polar* terms, although, of course we *can* take them to be ambiguous. At any rate, it would appear that terms with antithetical senses can generally be thought of as polar terms.

'Altus' connotes the property of being high or deep. But, as we have seen, this does not prove the term to be unambiguous, unless we are trying to prove that no term is ambiguous. Being either high or deep may be thought of as a disjunctive property (being high and being deep being its disjuncts). The question of ambiguity then becomes the question of whether the disjuncts of the complex property are sufficiently similar ("have something in common" — we feel, e.g., that scarlet and crimson satisfy this condition). It should be obvious that there is no clear-cut answer to this question. The answer we give will depend on many factors, some, as I have suggested above, linguistic in nature. Others are of a more pragmatic sort. By this I mean that if the objects subsumed under each of the disjuncts of a disjunctive property are such that we deal with them in a similar way (if they are not, to use Whorf's phrase, "operationally different"), we tend to feel that the properties *are* similar. Thus, to revert to our example, a Roman might well think of 'altus' as unambiguous in part at least because the problems of, say, traversing a high mountain and a deep ravine are similar in kind, being on the one hand the problems of ascent and descent, and, on the other, the problems of descent and ascent. I think this is sufficient to indicate what I mean here by *pragmatic* factors in determining ambiguity, and to afford at least a suggestion as to how consideration of such factors might be used to support the view that terms with antithetical senses are unambiguous.

That terms of the sort which Freud and Waismann mention are ambiguous is not, of course, a thesis for which either argues. Rather it is a point which each seems to take as obvious. I have undertaken only to question its obviousness.

ERRATUM

Owing to a proof-reading lapse for which the author apologizes two serious misprints are to be found in the article by Jørgen Jørgensen "Towards a theory of inference" (*Theoria*, Vol. XXV, 3, 1959). In order to get the author's meaning the readers are kindly asked to insert a "not" before the word "possible" in the third line on p. 135, and to substitute the word "designation" for the word "significance" in line 15-16 on p. 137.

Jørgen Jørgensen

Outline of a theory on the general logical structure of the language of action

by

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0. *Introduction*

A large number of diverse enquiries into the functions and nature of the language of norms and imperatives have been made, particularly in the last twenty years.¹ A good deal of clarification has been achieved. We know much more clearly than our ancestors what are the relations between imperatives and decisions, between principles and actions, between moral and non-moral rules, etc. But at this stage of analytic atomization we are perhaps on the verge of dissolving our understanding of the nature of that type of language into a big mass of unconnected "facts" or results. Thus, it is my aim in this short paper to make an effort at synopsis. I want to offer the outline of a view on the general logical structure of the language of action, in which the results so far obtained by analysis find their place and their mutual relationships become apparent. It is hoped that a global view of the language of action with its main divisions emphasized can present at least a different perspective of the intricacies and peculiarities of what used to be called practical reason.

1. *The field and subject-matter*

- 1.0. The subject-matter of the theory to be outlined in the following pages is the special type of thinking or of language exercised in deciding, entreating, requesting, advising, commanding, permitting, saying what somebody ought to do or avoid doing, and suchlike. These are the fundamental activities of human *praxis*, together with

¹ The bibliography at the end only lists a few of that large number.

blaming and praising. Since they involve the use of concepts or of language, they will be referred to as *varieties of prescriptive discourse*. In the sequel we shall not be concerned with the detailed differentiation among them, or with the possibility of reducing some of them to the remaining (say, e.g., entreating to requesting).

- 1.1. The exercise of any variety of prescriptive discourse involves the use of some special sentences or terms, those which are typical of the language of action and signal its use in contradistinction to other types of thinking or talking (e.g., the language of description, the language of esthetic or metaphysical contemplation). Thus, our field of study is constituted by all assertions or used sentences which may be determined as carriers of some variety of prescriptive discourse, i.e., as means to the exercise of such a variety. For instance, in quite ordinary contexts that the reader can supply with great ease, the utterance of the following sentences would constitute assertions belonging to the language of action or prescriptive discourse:

- (a) I will be there tomorrow [intended as a decision, not a prediction, as in fact is more often the case].
- (b1) Please pass the salt [at a dinnertable].
- (b2) Do it as quickly as possible.
- (c1) Yoy may make your first payment any day before July 30th.
- (c2) He ought to return that book, since it is overdue.

- 1.11. Clearly these sentences, or the assertions they are used to make, can be classified thus: (a) future-tense sentences which formulate a decision; (b) imperatives; (c) sentences with an auxiliary term like 'may', 'ought', 'should', 'right', 'wrong', which formulate duties, obligations, permissions, prohibitions, etc. We shall refer to (a) as *resolutives*, (b) as *imperatives*, and (c) as *normatives*.
- 1.12. We can make the term 'imperative' more precise for our purpose by defining an *imperative* as (1) any complete

sentence which is exclusively employed to tell somebody what to do, or (2) any combination of sentences one of whose components is an imperative. According to these definitions it is immaterial whether a given sentence is in what grammarians call the imperative mood or not. When uttered in normal contexts the following are imperatives: "Will you write by return post informing me whether you received the package", (b1), (b2), "Those taking the examination are to deposit all their books and notes on the table". The sentence "You ought to do A and B" is *not* an imperative in "If you ought to do both A and B, you ought to do A".

- 1.13. A *resolutive* is (1) any sentence which may be regarded merely as the answer to the (possible) question "What shall I do?" addressed to oneself; (2) any combination of sentences one of whose components is a resolutive. "I am decided to do it" is a resolutive in sense (1).
- 1.14. A *normative* is (1) any sentence containing a normative term ('ought', 'right', 'wrong', 'should', 'permissible', 'permitted', 'forbidden', 'must', 'may', 'cannot' or near-synonyms) such that either it or its denial can be correctly linked to the imperative mentioning the same action by terms like 'thus', 'so', 'therefore', 'since' or near-synonyms, and (2) combinations of sentences at least one of which is a normative. According to (1), "You cannot drive on the left [so, don't drive on the left]" and "You may turn here" are normatives. Clearly, the denial of the latter "It is not the case that you may turn here" can be supplemented without any absurdity by "so, don't turn here".
- 1.2. Imperatives have many properties, many of which have been examined with great care and ability by different writers.² I shall mention here only a few of the most important.
- 1.21. Imperatives are used to move a person to do a certain

² Particularly valuable are [1], [4], [7], [9], [10], [13], [21].

action, but their utterance or their apprehension are not causes for the agent's doing the action. They have something like an intention of causality, but it is part of their nature that their utterances do not become fully-fledged causes.³

- 1.22. One and the same imperative can be employed to discharge several varieties of prescriptive discourse. For instance, "Bring a chair" can be used with a requestive tone or in a commanding voice. This suggests that we can speak of a *complete imperative*, namely, one which contains a reference to the variety of prescriptive discourse it expresses.
- 1.23. Imperatives are essentially second-person sentences. But there is a use of proper third-person imperatives – in which there is an intermediary or delegating person.⁴
- 1.24. Imperatives are neither true nor false.
- 1.25. There seem to be imperative reasonings or inferences, for we commonly combine imperatives with factual sentences by means of inferential or explanatory words, as was remarked in 1.14 and is further shown by
 - (a) Stay here, *for* he is not ready yet.
 - (b) Open the window *because* it is too hot.
- 1.26. Moreover, as Hare has remarked,⁵ we use all logical connectives with imperatives:
 - (c) Get out *all* of you.
 - (d) *If* he is sick, call a doctor.
- 1.27. But the analysis of imperative reasonings requires that imperatives have a semantical property or value not only different, but also unrelated to the truth of the ordinary, indicative or factual sentence which describes the action mentioned in the imperative as an event in the world.

³ In this connection Cf., for instance [2]: p. 251; [16].

⁴ This use of third-person imperatives was called to my attention by both Professor Everett Hall and Dr. Robert Binkley.

⁵ [9]: 25.

⁶ A very incisive analysis of imperative inference is found in [13]; cf. also [7]: 140 ff.

Otherwise, we would have to regard (a) and (b) below as valid inferences:

- (a) The waiter (is bringing or) has brought the dinner;
So, waiter, bring the dinner.
- (b) If you read this book, come to see me;
Read this book;
So, come to see me.

Clearly, both are invalid, and (b) must be distinguished from (c):

- (c) If you read this book, come to see me;
you have read it;
Therefore, come to see me.

- 1.28. In general, we must be able to say of inferences involving imperatives that they are valid if and only if, assigning the value truth to all the factual premises and the value X to all the imperative premises is automatically to assign the value truth to the conclusion, if it is a factual sentence, or X if it is an imperative.
- 1.3. Resolutives have been studied much less than imperatives and normatives.⁷ This has not been too harmful, for they have many properties in common with imperatives.
- 1.31. Resolutives (as distinct from predictions) are used to express, or in the making of, decisions to perform an action. It may be said that they are also employed with an intention of causality, though in a sense different from that in which an imperative is so used. When an imperative is used it seems to be part of such a use that the utterance is somehow to participate in a causal chain of events leading to the agent's performance. On the other hand, a resolute is used and inserted in a causal process which the utterance of the resolute itself indicates to be leading to the speaker's performance.
- 1.32. A resolute like "I will do A" may express a decision

⁷ The best discussion in modern terms is contained in [2]: 256-260.

which relates to the use of the corresponding imperative "Do A" in one or another variety. Yet, there are "independent" uses of resolutes – i.e., uses which express the result of a deliberation with no previous command or advice or request, etc. Thus, it seems better to regard *deciding* as a separate variety of prescriptive discourse to which resolutes belong exclusively.

- 1.33. Resolutes are essentially first-person sentences.
- 1.34. It is probably incorrect to say that a resolute is true or false, unless we confuse it with a prediction or the factual assertion "I have decided to do A".
- 1.35. There seem to be resolute reasonings or inferences. Many of our decisions seem to be inferred from both the facts pertaining to the circumstances we are in and the goals we pursue.
- 1.36. We use all logical connectives in resolutes:
 - (a) I will *not* do A.
 - (b) *If* he comes, I will do A.
 - (c) I will do *everything* he tells me.
- 1.37. The analysis of resolute inferences requires that resolutes possess a semantical value or property not coordinated with the truth (or falsity) of the corresponding factual indicative. Cf. 1.27 and 1.28. We would not want to regard as valid the following possible inference:

Tomorrow I shall certainly hear my alarm clock [prediction].
Therefore, tomorrow I will hear my alarm clock [resolve].

- 1.38. An imperative inference cannot be translated into a resolute inference, nor is a resolute inference translatable into an imperative one. The following inferences are quite distinct and it is possible to have one without the other:

(a)	If <i>p</i> is the case,	Peter,	do A.
	<i>p</i> is the case.		
	<hr style="width: 50%; margin-left: 0;"/>		
	So, Peter, do A.		

(b) If *p* is the case, I (Peter) will do A.

P is the case.

So, I (Peter) will do A.

In consequence, (c) and (d) below do not seem valid in general terms:

(c) I (Peter) will do A.

So, Peter, do A.

(d) Peter, do A.

So, I (Peter) will do A

The reason why (a) and (b) are different is basically that a resolute is not a self-addressed imperative, even though 1.23 and 1.33 may suggest it. On the other hand, whatever properties (as the analogues of truth) a resolute and an imperative possess, which validates or invalidates resolute and imperative reasonings, such properties will always co-exist whenever "I (Peter) will do A" and "Peter, do A" are used together. Thus, we could say that (c) and (d) are valid, *whenever* the listeners employ the imperative or the agent makes the corresponding decision.

- 1.4. Normatives have a complex nature and very often have been assimilated to imperatives, particularly imperatives of commands.⁸ A few basic properties are mentioned here.
- 1.41. Sometimes second-person normatives are used with the "force" or intention of imperatives. Likewise, first-person normatives are used in the manner of resolutives.⁹ But there are normatives which are used neither as imperatives nor as resolutives, e.g., "You may do A"; and those which are so used may also be used without the causal intention of imperatives or resolutives, e.g.

⁸ Cf. [1]: 104 ff.; something of this is also found in [11]: 404 ff.

⁹ Cf. [11]: 412 ff.; [2]: 259; [16].

- (a) You ought to do it, but don't do it.
 (b) I ought to stop doing it, but I won't.
- 1.42. Normatives may be said to be true or false: "It is *true* that you ought not to do that."
- 1.43. Obviously, there are normative inferences or reasonings.
- 1.44. The truth-values of normatives are independent of the truth-values of their corresponding ordinary or descriptive indicatives. The following are clearly invalid:
- (a) He has come.

 So, he ought to have come.
- (b) He ought to come.

 Therefore, he will come.
- 1.45. Normatives seem to be rational means of guiding conduct in a sense in which imperatives or resolutes are not – for instance, they are not properly employed together with threats – they seem to indicate or suggest or presuppose that there are some (good) reasons for doing the actions they prescribe.¹⁰ Examples:
- (a) I will do it or he'll kill me.
 (b) Do it or I'll kill you.
 (c) You ought to do it or I'll kill you.
- 1.46. This is why "I ought to do it" is very often felt to be ground for "Do it" and "I will do it". Now, this may be put by saying that sometimes a normative implies its corresponding imperative or resolute.¹¹
- 1.47. Normatives are used, more than imperatives and resolutes, in connection with inferences or reasonings.

2. Step to the Theory

- 2.1. It is hoped that the reader will be satisfied that the above "facts" can be easily verified by an analysis of our ordi-

¹⁰ This is one of the main points in intuitionistic views, which has been greatly emphasized recently in [22]; particularly, pp. 28, 58.

¹¹ Cf. [4]: 15 ff.; [9]: 168 ff.

nary imperatives, resolutes, and normatives. So, these "facts" are to be explained by any theory of prescriptive discourse. Obviously, the list is far from exhaustive – many more can be found in the literature. Also, these "facts" are vaguely formulated; but this is part of the task. These "facts" (as well as others) can only be understood in precise terms when they are "located" in a theory. That is, the theory will provide a clarification of and a rigorous sense for these "facts" by determining their systematic relationships.

2.2. The theory to be outlined makes several assumptions, some of which are made explicit. It consists (as all theories) of a model and a series of commentaries.

2.21. First of all we assume that there are levels or isolable compartments of language (or thinking). The imperatives, resolutes, and normatives we are considering belong to an object-language, and there are properties they possess which in some sense can be expressed in that object-language, whereas others can only be thought of in the *meta-language* of that (object-)language.

2.22. We also assume that most of the properties of a system of sentences or language can be said to be (i) syntactical, (ii) pragmatic, and (iii) semantical. In our case, we can briefly say that:¹²

(i) *the syntactical dimension* is the totality of a sentence's purely formal properties and formal relations to other sentences, regardless of whether they have any function in the world or not.

(ii) *the pragmatic or applicative dimension* consists of the relations of prescriptive sentences to action, i.e., in what relates to the use of *some* prescriptive sentences in a manner which essentially includes that the utterance of those sentences is somehow connected with the agent's performing the action mentioned in them.

¹² Here we follow Professor W. Sellars' way of distinguishing these dimensions of language. Cf. [17]: 335 ff.; [18].

- (iii) *the semantic dimension* consists of those formal properties of prescriptive discourse which depend on the mere assumption that prescriptive sentences have the applicative dimension.

3. *The Syntax of Prescriptive Discourse*

- 3.0. We regard as the basic prescriptive sentence or clause an (artificial) incomplete sentence called *imperative-resolutive*.¹³ All other prescriptive sentences are elaborations of this. In ordinary English our imperative-resolutive appears as an infinitive or gerundial clause: "Peter's doing A" or "for Peter to do A", etc.
- 3.01. The most elementary contrast (in our theory) is that between ordinary descriptive sentences, e.g., "Peter did A", and imperative-resolutives.¹⁴ The fundamental syntactical difference is one of prediction or "mood". To make this apparent we shall represent indicative predication by means of square parentheses. For example:
- (a) Peter paid his rent – Peter (paid) his rent.
 (b) Peter, pay your rent – Peter [pays] his rent.¹⁵
 (c) I, Peter will pay my rent – I, Peter [will pay] my rent.¹⁵
- 3.02. To round the theory we shall enrich the imperative-resolutive "mood" or predication by introducing in it all the tenses we have in the descriptive indicative "mood". The question why in ordinary speech we only have present – (or future –) imperatives and resolutives belongs to the pragmatics of prescriptive discourse.¹⁶ Thus, we distinguish the following:
- (a) John [paid] his rent – John (paid) his rent.
 (b) John [will have paid] his rent – John (will have paid) his rent.

¹³ Cf. 1.3, 1.21, and 1.31 above.

¹⁴ Cf. 1.24 and 1.34, 1.41 and 1.45.

¹⁵ Here the right-hand side is not a complete sentence, where the left-hand side is, cf. 3.21 and 3.22 below.

¹⁶ Cf. (ii) in 2.22 and notice italics.

- 3.1. We regard normatives as (i) indicative in character, (ii) formed out of imperative-resolutives by means of the special normative words.¹⁷ Thus:
- (a) Peter ought to write an apology – Peter (ought) [to write] an apology.
 - (b) Mary is forbidden to go – Mary (is forbidden) [to go].
- 3.21. Imperatives, i.e., complete imperatives,¹⁸ are obtained from imperative-resolutives by means of an indicator of the variety of prescriptive discourse the imperative discharges. We shall refer to such indicators (which in ordinary English are words like 'please', 'would', 'I order', 'I command', 'I beg', 'I request', as well as special intonations) as the *pragmatic operators*, and for the sake of uniformity we can denote them by the adverbs 'advisingly', 'commandingly', 'requestingly', etc. Example:
- (a) Peter [come] here, commandingly.
- 3.22. Resolutives are obtained from imperative-resolutives by means of the indicator of decision.¹⁹ We may use the word 'decidingly' as the standard procedure. Here we have the *pragmatic requirement* that the user of the sentence must be mentioned in it as an agent:²⁰
- (a) I, John [do] A decidingly.
- 3.23. Normatives may also be formed out of (complete) imperatives or resolutives.²¹ So, a normative uttered in a commanding tone may be written as:
- (a) You (ought) [to do A], commandingly.
- And a normative expressing a resolve as:
- (a) I (ought) [to do A], decidingly.

¹⁷ Cf. 1.32 above.

¹⁸ Cf. 1.22.

¹⁹ Cf. 1.32.

²⁰ Cf. 1.33.

²¹ Cf. first part of 1.41.

- 3.3. In the case of compound sentences we regard imperatives as the more dominant. Next come resolutes. If an indicative compound includes a normative component, the indicative is normative. All (truth-functional) connectives 'and', 'or', 'if-then', etc. have the same meaning in both imperative and indicative compounds.²²
- 3.31. The negation of a normative sentence produces the elimination of all pragmatic operators in the normative.²³ Thus, from "I (ought) [to do A] decidedly" we obtain "I (may) [leave A undone]" or "It is not the case that I (ought) [to do A]".
- 3.4. Imperative-resolutes are inferred in accordance with the customary rules governing ordinary descriptive indicatives.
- 3.41. The pragmatic operators may be disregarded in examining the validity of any imperative or resolute inference. But it should be remembered that no single imperative-resolute can be affected by two (different) pragmatic operators. We shall regard as valid the inferences in which the decision operator is substituted for or by another pragmatic operator, as in
- (a) Peter [do A] commandingly.
- So, I, Peter [do A], decidedly.
- 3.42. The rules governing normative inferences can be reduced to a simple set, as is shown in 3.45, but the reader may skip this and go straight to sections 4 and 5. Here

²² Cf. 1.26, 1.36. In this respect I differ from Prof. E. Hall, [7]: 126 ff. I agree with Mr. Hare (who has not considered mixed indicative-imperative compounds) that the connectives 'or', 'and', etc. have always the same meaning. Yet Prof. Hall is correct in holding against Mr. Hare that "Put on your parachute and jump out" does *not* in its ordinary sense entail "Jump out". The first imperative is just short for "Put on your parachute and then (i.e., if you *have* put your parachute on, then) jump out". Indicatives of the form "*p* and if *p*, then *q*" do entail *q*; but the above imperative inference requires the indicative "you have put on your parachute" as an additional premise. Cf. [7]: 142.

²³ Cf. second part of 1.41.

are some illustrations of those which may be regarded as more fundamental.

- 3.421. If an imperative-resolutive prescribes a tautological act, then the corresponding 'ought' – or 'obligatory' – sentence is also tautologous.

(a) Peter (ought) [to go or stay].²⁴

- 3.422. Two examples of a rule of valid inference:²⁵

(a) Peter (ought) [to do A].

So, it is not the case that Peter (ought) [to leave A undone].

(b) Peter (ought) [to do A] commandingly.

So, Peter (may) [do A].

- 3.423. One example of another rule:²⁶

(a) Peter (ought) [to do A or B].

So, Peter (may) [do A] or he (ought) [to do B].

- 3.424. One half-example of another rule:²⁷

(a) He (ought) [to pick up] his wife at Smith's.

Therefore, his wife (is) at Smith's.

- 3.425. "If everybody (is obliged) [to do A], then it (is obligatory) that everybody [do A]."²⁸

- 3.426. Examples of the relation between normatives and imperatives or resolutes:²⁹

(a) If you (ought) [to do A], [do A].

(b) If I (ought) [to do A], I [will do A].

²⁴ This is an example of rule RIII of the system M*, Cf. *3.533.

²⁵ This rule is derivable from Axiom A50 of *3.533.

²⁶ This rule is derivable from A51 of *3.533.

²⁷ This rule is derivable from T1 of *3.55, which comes from A52 of *3.533.

²⁸ This is an example of A54 of *3.533.

²⁹ These are examples of A53 of *3.533. Mr. Hare has made (a) part of his definition of normative sentences; Cf. [9]: 164 ff.

*3.5. *An Axiomatization of Prescriptive Logic*

This is a formal treatment of the syntax of prescriptive discourse, but it may be skipped by the reader.³⁰ We outline here a formal system to be called M^* .³¹

- *3.50. We assume the usual sets of primitive terms (for individual constants and variables, predicates, connectives), but we assume two new kinds of signs: (a) pragmatic operators: ' δ ' (for 'decidingly'), ' φ ' (for 'commandingly'), ' ψ ' (for 'requestingly'), etc.; and (b) prescriptive connectives: ' I ' (which transforms an imperative-resolutive into the corresponding descriptive indicative) and ' K ' (which forms a normative when prefixed to an imperative-resolutive, and should be read as 'ought' or 'it is obligatory that ...'). Examples:

Peter gives John the book --- Give (Peter, John, the book)

I command, commandingly,

Peter, give John the book --- φ (Give [Peter, John, the book])

I Peter decide to, will do A --- δ (A [Peter]).

Peter ought to do A --- K (A [Peter]).

- *3.51. We assume the customary rules of formation for the primitive ordinary connectives (which we should like then to be negation and disjunction or conjunction) and quantifiers, but now they are also utilized to form imperative-resolutives, when one component is an imperative-resolutive.

- *3.511. The rules for atomic sentences are:

(1) The sentences of the form $Z_n(X_1, X_2, \dots, X_n)$ are

³⁰ For other formal treatments, or proposal for formal treatments, of imperatives, Cf. [3], [10], [23]–[26]; and for treatments of normatives Cf. [3], [9]: 188–197, and [27]; in [6] the author formulates legal principles which are valid more generally, i.e., even for non-legal rules.

³¹ I have chosen this name simply because 'M' is the initial of my wife's name 'Miriam'.

indicatives, where Z_n is an n -adic predicate and the X_i are individual signs.

- (2) The sentences of the form $Z_n[X_1, X_2, \dots, X_n]$ are imperative-resolutives, where Z_n and the X_i as in (1).

*3.512. The formation rules for the new signs are:

- (1) The sentences of the forms $K(Z)$ and $I(Z)$ are indicatives, where Z is an imperative-resolutive.
- (2) The sentences of the form $R(Z)$ are imperative-resolutives, provided R is a pragmatic operator and Z is an imperative-resolutive which has no pragmatic operator at all, or has at least one imperative-resolutive component with no pragmatic operators and all others with ' δ '.

*3.52. We accept the customary definitions of 'bound variable', 'theorem', etc., as well as those for the defined connectives and quantifier, which are now generalized to include imperative-resolutives. But the additional definitions are needed:

- (1) $P(Z)$ as a short for $\sim K(\sim Z)$.
 "Z is permissible" or "Z may be performed" is a short for "It is not the case that omitting Z is obligatory".
- (2) $F(Z)$ as a short for $K(\sim Z)$.
 "Z is forbidden" or "It is wrong to do Z" is a short for "Z ought not to be done".
- (3) $L(Z)$ as a short for $\sim (K(Z) \cdot \sim (K(\sim Z)))$.
 "Z is completely optional" or "Z is completely up to you" is a short for "Neither the doing nor the omitting of Z are obligatory".

3.53. M^ includes, first, an imperative-resolutive generalization of the appropriate

- (1) Axioms for general propositional logic;
- (2) Axioms for general quantificational logic.

*3.531. The axioms for the basic imperative-resolutive logic are:

$$A31. I(Z_n[X_1, \dots, X_n]) \equiv Z_n(X_1, \dots, X_n).$$

$$A32. I(\sim Z) \equiv \sim I(Z).$$

$$A33. I(Z \vee Y) \equiv I(Z) \vee I(Y).$$

$$A34. I(Z \vee Y) \equiv I(Z) \vee Y.$$

$$A35. I(Y \vee Z) \equiv Y \vee I(Z).$$

$$A36. I((X)(Z)) \equiv (X)(I(Z)).$$

Here Z_n is an n -adic predicate, the X_i are individual signs, X is an individual variable, Z is an imperative-resolutive, and so is Y in A33; but Y is an indicative in A34, A35.

*3.532. The axioms for the general logic of the varieties of prescriptive discourse are:

$$A41. R(\sim Z) \equiv \sim R(Z).$$

$$A42. R(Z \vee Y) \equiv R(Z) \vee R(Y).$$

$$A43. R(\delta(Z) \vee Y) \equiv \delta(Z) \vee R(Y).$$

$$A44. R(Z \vee T) \equiv R(Z) \vee T.$$

$$A45. R(T \vee Z) \equiv T \vee R(Z).$$

$$A46. R((X)(Z)) \equiv (X)(R(Z)).$$

Where R is a pragmatic operator, Z and Y are imperative-resolutives, T is an indicative, and X is an individual variable.

*3.533. The axioms for general normative logic are:

$$A50. K(Z) \supset \sim K(\sim Z).$$

$$A51. K(Z \supset Y) \supset (K(Z) \supset K(Y)).$$

$$A52. K(T \cdot (Z \vee \sim Z)) \equiv T.$$

$$A53. K(Z) \supset Z.$$

$$A54. (X)(K(Z)) \supset K((X)(Z)).$$

The symbols are as in *3.532.

Examples:

A50. (a) "If Peter ought to keep that promise, then it is not wrong for him not to keep it."

(b) "If it is obligatory to drive on the right, it is not the case that one is obligated not to drive on the left."

A51. (a) "If it is obligatory that (on doing A I do B), then if A is obligatory for me B is also obligatory for me."

In our system M^* we distinguish between $K(p \supset Z)$ and $K(Y \supset Z)$, where p is a factual statement and Y and Z are imperatives. The former can be proved in M^* to be logically equivalent to $p \supset KZ$ (cf. *3.55), but the latter cannot be proved equivalent to $KY \supset KZ$, even though it entails this (in accordance with A51).

A52. (a) "It is obligatory that you who are sick either pass or not pass judgment on him, if and only if you are sick."

A53. (a) "If you ought to keep your promise, keep it."

(b) "If I ought to seem, then I will see him."

3.54. The rules of inference within M^ are:

RI. *Modus ponens*: From a formula Y and a formula $Y \supset Z$, one may derive Z .

RII. *Rule of universalization*: From a formula Z one can derive $(X)Z$, where X is a variable and both X and Z satisfy certain conditions depending on the axioms for general quantificational logic.

RIII. *Rule for normative inference*: If an imperative-resolutive formula Z is a theorem of M^* , then $K(Z)$ is also a theorem of M^* .

It should be noticed that RI and RII constitute an imperative-resolutive generalization of the customary rules of those names.

*3.55. From the preceding rules and axioms we can derive an imperative-resolutive calculus, which parallels the ordinary propositional and functional calculus. By means of RII we can derive theorems like the following: Let ' p ' be an indicative, and ' Z ' and ' Y ' imperative-resolutives:

T1. $p \cdot KZ \equiv K(p \cdot Z)$.

T2. $p \vee KZ \equiv K(p \vee Z)$.

Examples of T1 are:

(a) "This is your friend and you ought to say 'hello' to him, if and only if you ought to say 'hello' to this friend of yours";

- (b) "There is a chair in the next room and you must bring it here, if and only if you must bring a chair which is in the next room."

T3. $p \supset KZ \equiv K(p \supset Z)$.

T4. $L(Z \cdot Y) \supset (LZ \cdot PY) \vee (LY \cdot PZ)$.

Examples of T4 are:

- (a) "If going for a walk and stopping at the airport is something one can freely choose to do or not to do, then either going for a walk is quite up to him while stopping at the airport is allowed or going for a walk is just permitted while stopping at the airport is something one can freely choose to do or not to do";
- (b) "If one can do as he pleases concerning actions A and B, then either A is permitted [perhaps obligatory] but one can do as he pleases concerning B, or vice versa."

T5. $KZ \supset PZ$. "Whatever is obligatory is allowed (or permitted)."

T6. $FZ \supset F(Z \cdot Y)$. "If an act is forbidden it is forbidden to do it together with any other act whatever."

T7. $P(Z \cdot Y) \supset PZ \cdot PY$. "If it is permitted to do two acts together, then each one is itself permitted."

T8. $F(Y \vee Z) \equiv FY \cdot FZ$. "It is forbidden to choose between two given acts, if and only if each act is forbidden by itself."

T9. $FY \vee FZ \supset F(Y \cdot Z)$. "If one of two acts is by itself forbidden, then it is forbidden to do them together."

T10. $LZ \equiv L - Z$. "A given act is completely optional, if and only if its omission is completely optional, too."

T11. $(p \supset q) \cdot K(q \supset Z) \supset K(p \supset Z)$.

Example:

"If mailing the letter is a sufficient condition for

his receiving it and you are obligated to pay him on his receiving the letter, then you are obligated to pay on mailing it."

4. *The semantics of prescriptive discourse*

- 4.1. In ordinary language we lack general words like 'truth' and 'falsity' to refer to the values possessed by imperatives and resolutes. This is due to the fact that indicative assertions either i) correspond to some fact, which makes them or their negations false, or ii) are organized along the pattern of factual discourse so that there are some sort of object to which they ascribe properties. On the contrary, imperatives and resolutes are just sentences uttered in connection with moving some person or persons to action. Our interests in using imperatives and resolutes are more variegated. But some semantical values are required to formulate a theory of the logical validity of inferences involving either imperatives or resolutes. As remarked in 1.27 and 1.28, 1.37 and 1.38, it is impossible to explain the validity of an imperative or resolute inference by saying that the corresponding indicative inference is valid, i.e., the inference obtained from the former by replacing every single imperative or resolute by its corresponding indicative. Once again, such an explanation would require that argument (a) below be valid:

- (a) If you read this book, come to see me;
 Read this book;
 So, come to see me.

For its corresponding indicative argument (b) below is certainly valid:

- (b) If you (have) read this book, you will come to see me;
 You have read this book;
 So, you will come to see me.

Yet, (a) is invalid: issuing the injunction to read the book and the conditional injunction to see me on reading it is *not* to issue the injunction to see me (*simpliciter*).^{31a} (Of course, there is a use of 'you will come to see me' in which it formulates an order; but here it is intended as formulating a prediction, i.e., as a statement of fact.)

- 4.11. The semantical values of imperatives and resolutes we are after must be such that: (i) they can be resorted to in discussing the validity of an argument containing imperatives or resolutes; (ii) they are somehow related to the action to be performed, but not to the truth of the indicative formulating such an action as an actualized or possible event; (iii) they relate to circumstances of both the speaker and the possible agent, rather than to the mere intentions of the former. One of such values is such that, it is presupposed in the employment of imperatives in language, in normal circumstances once the hearer recognizes that it is possessed by any imperative premises of the argument it (a) will be possessed also by the imperative conclusion, if the argument is valid, and (b) will elicit his willingness to perform, insofar as he is a rational agent. In other words, one of the values of the imperatives involved in imperative inferences is such that if it is possessed by an imperative, then action in accordance with this imperative, as obedience to it, will be reasonable or justified.
- 4.12. Imperatives have the semantical values *justified* and *non-justified*. (The words are awkward; but we are interested in the concept anyway.) In ordinary language we say that an order or command or request or piece of advice is reasonable, justified, correct, appropriate, proper, convenient, etc. or not. These words are not all the right ones in a given context. But there is a common thread

^{31a} Hare's proposed values *assent* and *dissent* to check the validity of an argument would make (a) valid, for one assents to an imperative by obeying it (See 9, 19 f.), i.e., if the corresponding imperative is true. Thus, since (b) is valid, (a) is valid too.

running through all of them. That is the feature on which I want to concentrate my attention. These words are not completely interchangeable in the English idiom, and that shows that there is nothing like a fact corresponding to them so that we could say univocally and simply "the imperative . . . is justified" in the way we can say that the statement or the indicative — — — is true (or false).

An imperative is justified in one context with respect to certain purposes or aims, but it may be non-justified with respect to other purposes. There are many semantical values of imperatives justifiedness and non-justifiedness. Whereas the truth or falsity of an indicative sentence with a precisely specified meaning, or a statement, is one.

Exactly the same holds for resolutes.

- 4.13. Let us fix a context *C*, i.e., a set of ends *E* which are pursued by both the speaker and agent *X* together with their logical and causal connections to the circumstances of the speaker and *X*. Now we can define:

- (D1) The imperative "*X*, do *A*" is *justified in C* if and only if (i) Both *X* and the speaker subscribe to *E*, (ii) *X*'s doing of *A* is such that at least it does not further the failure of *E*, and (iii) *A* conforms to the conventions and decisions to which both *X* and the agent subscribe in their pursuance of *E*.
- (D2) The imperative "*X*, do *A*" is *non-justified in C* if and only if it is not justified in *C*.
- (D3) The resolute "*I(X)* shall do *A*" is *justified in C* if and only if (i) *X* subscribes to ends *E*, (ii) *X*'s doing *A* will not prevent the attainment of *E*, and (iii) *A* is logically consistent with *X*'s accepted conventions and decisions for the attainment of *E*.
- (D4) The resolute "*I(X)* shall do *A*" is *nonjustified in C* if and only if it is not justified in *C*.

- 4.14. The values *justified in C* and *nonjustified in C* determine the structure of deliberation, be it oriented toward one's

decision or toward a reasoned commanding or advising. But the assertion of a resolution or a command (advice, request, etc.) requires that the speaker expresses that the decision or command be justified (*simpliciter*). To say "Please pass the salt" or "But, bring the book next Monday" is to imply (in Moore's sense) that the hearer's passing the salt or bringing the book next Monday is not merely justifiedly asked in some context C, but that it is justifiedly asked regardless of whatever else happens. Of course, to assert an imperative or a resolute is not to assert, or even to imply, that one has in fact gone through a process of examination of all possible consequences of the action in question and how it relates to the circumstances of every person however remotely affected by it. In those cases the speaker is acquainted with the most important facts of the situation and he just "sees" that very likely nothing else matters.

- 4.15. The different ends a person pursues fall within a rough hierarchy. Some are not only subordinates in the sense that they are parts of more comprehensive ends, but are subordinate in the sense that they are less important. In the case of a conflict the one which is sacrificed is lower. Clearly, nobody is aware of a sharply drawn series of boxes containing his precisely formulated ends, so that he can just "look" at it and know which one is higher. Indeed, one comes to know that a given end is higher only because on the occasion of a conflict he choose to favor it. The hierarchy of ends is something vague and constructed along one's march on the path of life. It is winding path, and often one destroys his previous construction, however careful and difficult its building was.

An imperative or a resolute is used to make an assertion against the vague background of the hierarchy of ends and conventions the speaker subscribes to and believes that the agent also does. In deliberation the speaker does try to run through the whole series of steps of the hierarchy, and even try to walk slowly while climb-

ing some of them. Most often he just realizes, or claims to have seen, that nothing conflicts with the highest ends in question.

An imperative or resolutive is *justified (simpliciter)* if the context C is made up by the highest ends the speaker and the agent subscribe to.

- 4.151. In the case of ordinary human beings the context made up by their highest ends includes the ideal of morality, i.e., the ideal that we are entitled to equal consideration for sharing the existing goods and conditions of our world. But they do not logically have to accept the ideal of morality; it is only a fact that ordinarily they cannot afford not to accept morality.
- 4.20. Normatives are of the indicative family. They are not descriptive of facts, but they are true, or false, and there is some sort of fact that they formulate. It is not an empirical fact, or a mathematical fact, which is entailed by the mere existence of objects. Normatives describe pseudo-facts, namely, facts of logic: Facts which belong to the logical structure of the basic prescriptive discourse, the discourse of imperatives and resolutives. These are basic in the sense that they are exclusively concerned with the rational production of action.
- 4.21. The fundamental semantical property of normatives is exhibited in the following formulas:
 - (a) If X ought to do A, then the imperative-resolutive schema "X [does] A" is justified.
 - (b) If the imperative "X, do A, commandingly" is justified, then X ought to do A (commandingly).
- 4.22. In general, a normative of the form "A is obligatory" or "A ought to be performed" is an object-linguistic sentence which *in* its meta-language (i.e., when the meta-language includes its object-language) both entails and is entailed (i) by a meta-linguistic sentence which says that the imperative-resolutive mentioning A is necessarily justified, and (ii) by a meta-linguistic sentence which

says that the imperative-resolutive prescribing the non-performance of A is necessarily nonjustified.⁸²

A normative of the form "A may be done" or "A is permitted" is a sentence in the object-language which in the meta-language both entails and is entailed by a meta-linguistic sentence which says that the imperative-resolutive prescribing the non-performance of A is not necessarily justified.

A normative of the form "A is forbidden" or "A ought to be omitted" is an object-linguistic sentence which both entails and is entailed by the meta-linguistic sentence which says that the imperative resolutive schema mentioning A is necessarily non-justified.

- 4.23. Normative language is the reflection on the extended object-language of imperatives and resolutes of that part of the meta-language in which we talk about the justifiedness-values of imperatives and resolutes. It includes the picture of these values as properties of actions, as when we say "action A is right (wrong), or ought to be done", as well as a secondary picture of those semantical values as properties of objects or persons, as when we say "This is the right thing on which . . ." or "This is the one which must . . ., or who ought to . . .". In Carnap's old terminology, the language of norms is the material mode of speech of the syntactic-semantical language of the justification of imperatives and resolutes.
- 4.3. As a result of 4.21 and 3.426, and 3.41, we have the following valid inferences:

(a) The imperative "Peter do A" is justified.

Therefore, Peter do A.

(b) The imperative (or command, or request, etc.)
"Smith, do A" is justified.

So, Smith, do A (commandingly, requestingly, etc.).

⁸² Cf. 1.45 and 1.46 above.

- (c) The imperative (or command, etc.) "Smith, do A" is justified.

So, I, Smith will do A (decidingly).

5. *The functions of prescriptive discourse*

- 5.1. The characteristic feature of prescriptive discourse in its applicative dimension is its involvement with action. And in this connection we shall refer to the two points below as *Sellars' pragmatic thesis about prescriptive discourse*.⁸³
- (S1) If a language L is prescriptive, then there are some sentences S_i in L such that the thoughts in which any of the sentences S_i are employed are motivational.
- (S2) The thought which involves the use of an action-sentence S is motivational if and only if it brings about a tendency in the thinker-agent to do the action mentioned in S.
- 5.11. By (S2) we do not intend to say that a motivational sentence is used in a thought which produces a new fresh motive. We simply mean that the thought employing that sentence produces an organization of the agent's motives in such a fashion that those motives urging the performance of the action mentioned in the sentence come to be more dominant. In 'performance of an action' we also include the (performance of the) omission of an action.
- 5.12. Our view here is that the basic motivational sentences are imperatives and resolutives.
- 5.121. To understand an imperative addressed to him, a person has to think in a way which involves the imperative; and a tendency or quasi-tendency to do an action which has been, say, commanded is, or may be, brought about by the (understanding) thought of the command.
- 5.122. Resolutives are not mere *ex post facto* reports or expressions of previously made decisions. In our definition we

⁸³ Cf. [16] and [19].

explicitly excluded any role as a report which our resolutives may have *in addition to* their use *in* making and *to* make decisions, by being involved in thoughts which bring about a re-arrangement of motives.³⁴

- 5.123. It is perhaps true that in some varieties of prescriptive discourse, like advising, and entreating, as contrasted with commanding, for instance, the imperatives used in them are not directly motivational, that they are so only through the medium of an intervening resolute, when and if such a resolute is used. Perhaps this is true of all imperatives³⁵
- 5.13. Some normatives are also motivational, but on the view put forward here they are motivational in a derivative sense, *via* imperatives and resolutives.
- 5.2. Possibly, prescriptive discourse ultimately rests on the blind conditioning that evokes certain responses to certain utterances, a process which is still to be found in parade drills. But the essence of prescriptive discourse lies in constituting the instrument for rational action.
- 5.21. An elementary motivational thought involving an imperative is one in which, leaving aside conditioned responses, (i) the agent-thinker is acquainted with the imperative and understands it, and (ii) he believes or is aware that the circumstances are of such and such a kind, which make the imperative a justified one, but (iii) he does not, or does not have to, believe that the imperative is justified.³⁶ This we may call *acting on an imperative* in contrast with *acting on the justifiedness of an imperative*, which includes (i), (ii), (iv) the agent's realization or belief that the imperative is justified, and (v) and the (mental or verbal) move from "(the order, request, advice, etc.) to do A is justified" to the resolute "I will do A".³⁷ The second type of acting is beyond

³⁴ Cf. 1.31 above.

³⁵ Cf. 3.41 above.

³⁶ Cf. 1.21 above.

³⁷ Cf. 4.3, particularly (c).

our present program – it belongs to the pragmatics of meta-languages.³⁸

5.22. The same points apply, *mutatis mutandis*, to the difference between *elementary deciding* and *deciding on the justifiedness of the decision*.

5.3. *To act on a normative* like “A ought to be done (or omitted)” is somehow between (1) acting on an imperative or deciding in the primary sense and (2) acting on the justifiedness of the command, request, etc. or decision. *In a way* it is acting on the justifiedness of the decision, or command, etc. In this case the awareness or belief that the command, request, etc. is justified is somewhat oblique; the sentence becomes transparent, so to speak, and the justifiedness value appears directly attached to the action mentioned in the sentence. The thought that an action A ought to be done (or omitted) by me, or is convenient (proper, adequate, etc.) to me, involves a gleaning of, but it is not the thought of, the justifiedness of the advice, or command, etc. (possibly given me), or of my decision, to do A.³⁹

5.31. A first-person normative is used motivationally if and only if it contains a (complete) resolute.⁴⁰ Thus, when I assert that I ought to do A with a deciding or resolute intonation, I am (i) making the decision to do A, (ii) expressing without asserting it that my decision is justified, and (iii) evincing my willingness to do the action *as* if it had been prescribed in an imperative of command, advice, etc. A normative which satisfies (1)–(iii) will be said to be an *Achtung*-normative; for that seems to be the idea Kant was driving at in his conception of *Achtung*.⁴¹

³⁸ Cf. Sellars, [19].

³⁹ Cf. 1.45 above.

⁴⁰ Cf. 3.22 as well as 3.21.

⁴¹ Cf. for example: “Was ich mich unmittelbar als Gesetz für mich erkenne, erkenne ich mit Achtung ... Die unmittelbare Bestimmung des Willens durchs Gesetz und das Bewusstsein derselben heisst Achtung.” ([11]: 402 n). Kant referred to moral norms only, not to all norms, cf. 6.3 below.

- 5.32. Second-person normatives are motivational if and only if they contain a (complete) imperative.⁴⁰ And their analysis from the point of view of the agent-listener is as is indicated in 5.3. From the point of view of the speaker parallel comments should be made. There is an elementary use of imperatives which consists in the speaker's reacting directly to his perception of certain circumstances by uttering an imperative in the appropriate variety of prescriptive discourse. Upon this use we can build a meta-linguistic use which mentions the justifiedness of the imperatives, i.e., of the orders, requests, entreaties, etc. Once again, the use of normatives is somehow in between the two – it points to the justifiedness of the command, request, etc. by saying that the action mentioned in the command, request, etc. is justified, convenient, proper, or due.
- 5.33. The same applies to third-person imperatives, *mutatis mutandis*.
- 5.4. Since resolutives and imperatives are in a way equivalent,⁴² the use of a motivational second- or third-person normative includes a hint not an assertion of the willingness of the speaker to choose to do the action mentioned in the normative, were he in exactly the same circumstances. This is the counterpart of the component (iii) of the use of first-person normatives. We shall embody the two under the label *approval*. We shall say that the speaker's approval is his attitude toward the agent's doing the action mentioned in the normative or imperative or resolute.
- 5.41. We shall say that a sentence *S* *phrastically implies* another sentence *S'* if and only if, giving the general conventions of the type of language to which *S* belongs, the use of *S* in (a) normal circumstances or contexts and (b) in the normal or central mode of the type to which *S* belongs, is impossible if *S'* is not true or justified, etc.

⁴² Cf. 3.0, 3.2, and 3.41.

depending on the kind of sentence S' is. By "normal contexts" we mean circumstances in which the speaker is aware of what he is saying, and he is not lying, etc. By "normal or central mode of use of a sentence" we refer, e.g., to the assertion of an indicative in contradistinction to the use of it as an unasserted component in a larger sentence, to the use of it in *just* considering the situation or state of affairs it denotes, etc. Clearly, there is a corresponding central mode of using an imperative,⁴³ an optative, etc., even if there is no general word in ordinary language which covers this use in advising, requesting, commanding, etc.

- 5.42. We shall say that a normative of injunction, i.e., one of the form "A ought to be done", phrastically implies that the speaker approves of the agent's doing A; that a normative of prohibition, i.e., of the form "A ought not to be done" phrastically implies that the speaker disapproves of the agent's performing A; that a normative of permission, i.e., of the form "A may be done" phrastically implies that the speaker neither approves nor disapproves of the agent's performing A.⁴⁴
- 5.43. The use of a normative of permission serves purposes which cannot be accomplished by the use of any imperative. Part of the sense of a normative like "You may do A" is expressed by "Do A, or don't do A"; but there is a difference – the normative informs of the value possessed by the imperative "Do A".⁴⁵ Moreover, the normative express, or phrastically implies the absence of disapproval of the agent's doing A.
- 5.5. The normatives which do not contain a (complete) resolute or imperative are not motivational. We can talk about the justifiedness-values of imperatives or of actions without committing ourselves to act.
- 5.51. Inferences like the following are formally invalid:

⁴³ Cf. [9]: 19 ff.

⁴⁴ This is the insight found in subjectivism.

⁴⁵ This is written thanks to a suggestion from Mr. Robert Binkley.

(a) I (ought) to do A.

So, I [do A] decidedly.

(b) You (ought) to do A.

So, you [do A] commandingly.

(a) and (b) are invalid because the resolute or imperative conclusion includes a pragmatic operator not contained in the premises. But since an imperative or resolute without a pragmatic operator is *not* a sentence, we cannot *actually* make or think an inference like (c), which is formally valid

(c) You, I (ought) [to do A].

So, you, I [do A].

For in the normal use of language we have to use complete sentences.

- 5.52. We shall say that inferences like (a) and (b) in 5.51 are *ellectically valid*, or that a normative *ellectically implies* its corresponding complete imperative or resolute, to mean that these inferences are validated because of the pragmatic requirement that actual speech or thinking proceeds in units which are complete sentences; i.e., whenever we have inferences like (c) we can only express them as in (a) or (b).⁴⁶

The same applies to

(d) "Peter [writes] an apology" is justified.

Therefore, Peter [writes] an apology requestingly.

6. Epilogue: Moral systems

- 6.1. Everything discussed in the three preceding chapters holds in particular for moral norms, imperatives, and resolutions.
- 6.2. The distinctive marks of moral norms lie (i) *inter alia* in

⁴⁶ These inferences are validated *in* the making of i.e., in *thinking* or *saying* them. The verb $\epsilon\upsilon + \lambda\acute{\epsilon}\delta\omicron\omega = \epsilon\lambda\lambda\acute{\epsilon}\delta\omicron\omega$ seems not to have existed in Classical Greek, but I hope the meaning of 'ellectic' as explained is clear enough.

its comprehensive character, i.e., in its providing a criterion for judging in principle not only all other actions judged within all other normative systems, but also for judging all other normative systems themselves; and (ii) in the specific "point" or end that is achieved by acting morally.⁴⁷

- 6.3. The motivational normatives of morality are (at least normally) the fundamental *Achtung*-sentences. Perhaps the continuous exercise of moral discourse is such, with its organization and strengthening of the motives favorable to the doing of the action we think that we (morally) ought to do, that moral normatives come to acquire a new role and begin to function as the originators of a motive. This is what is customarily called *sense of duty* or *conscience*.

But here we can only mention this as a possible correction to our discussion of what is generally involved in acting on a normative.⁴⁸

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Imperatives and orders

by

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At a very early age we learn to use gestures and linguistic expressions so as to influence the behavior of those around us. We learn, also, to respond properly to gestures and expressions which are used to direct our behavior. The gesture may be as simple as a mother's frown; the linguistic expression, 'give me some more!'.

In recent years such directive language has attracted the attention of logicians, moral, and legal philosophers. But their analyses have until very recently been of a restricted nature. The predominant concern has been to distinguish the class of imperatives as a whole from the class of indicatives as a whole. We learn to use words like 'order', 'advice', 'warning', 'plea'; we properly employ these words and understand what others mean when they employ such words. The variety of imperative uses represented by these words has rarely been analysed. Few philosophers have made explicit the principles underlying the use of gestures and sentences for various imperative purposes.

In this paper I have a three-fold aim. First, to set forth criteria for classifying an expression as an imperative one. Second, to examine with some thoroughness a particular imperative use of language of special interest in political and legal philosophy, namely, ordering persons to act. Thirdly, to suggest inadequacies in imperative theories of law. Such inadequacies have long been public knowledge but I believe one comes at them more advantageously through the analysis I present.

When the phrase 'imperative expression' is used by me, it means any gesture (it might be pointing with a finger or lifting an eyebrow) or linguistic expression (it might be 'stop!', 'you will stop!', 'will you stop!', 'I want you to stop!') which is being em-

ployed by one person or several persons to tell another person or persons or any thing thought capable of understanding to do or not to do an act or acts. For convenience, the phrase 'tell to do an act' will be used to refer to an expression so used and reference will not be made explicitly to negative imperatives and only occasionally will reference be made to several acts. Silence may operate as an imperative but for purposes of this paper it will be disregarded. The phrase 'imperative mood' means the standard grammatical structure or structures in a language for conveying imperative expressions. This mood is not a conclusive indication of the use we are making of a sentence. However, a particular mood is conventionally associated with a different function of language. The mood employed is some indication that a sentence is being used in one way rather than in another and context usually aids us in discerning an intention to deviate from a standard employment. The word 'act' means any voluntary physical or mental activity. I shall leave this matter in just so indefinite a form. In the discussion which follows, imperatives will be limited to those expressions used to tell 'another person' to do an act. It should not be inferred that one cannot issue an imperative to animals other than humans or to physical objects thought capable of understanding.

What are the criteria for judging whether an expression is an imperative one or not? It is a characteristic of imperative expressions that they mean something. But in a certain sense of 'mean' my red nose might mean that I have a cold, or my college tie might mean that I am, or have been, a member of the college. Smoke might mean fire and a yellow complexion might mean jaundice. A red nose, smoke, a yellow complexion indicate for us, because we have certain relevant knowledge concerning causal relationships, that something is the case. But it is also the case that persons use, among other things, gestures and words to mean something. And we can say of their use that they 'meant' something by their use. It is characteristic of imperative expressions that we can say that we 'meant' something by their use. 'When I raised my sword, that meant, "charge!".' We do not use a red nose or a yellow complexion, except on rare play-acting occasions,

to mean anything. Now, when we speak of our having 'meant' something by an expression, it is necessary to introduce the notion of 'an intention' in the use of the expression if we are going to elucidate the meaning of 'having meant something by'. The smoke or the college tie might be used with an intention, but in the sense that smoke 'means' fire, it is not so used. However, it is notorious that American Indian smoke signals are used with an intention; they do mean something; and one can go further and say of their use that someone 'meant' something by using them.

An intention of some sort, then, is related to an imperative use of language. What sort of intention must it be? A good case can be made out that it must be an intention that the person addressed do an act. Only then should we regard it as appropriate to say, 'by that expression, I meant him to do an act'. If you point at the door and say to me, 'there is the door', and if you intend simply to make me aware of what constitutes a door, your gesture and words are not being used by you imperatively. To the dentist drilling your tooth, you may say, 'it hurts'. If your intention is only to convey information, your expression is not used by you imperatively. It is not used by you to tell him to do anything. You may cry 'ouch!'. Here, if the requisite imperative intention is absent, language, if so it is to be regarded, is quite generally speaking and perhaps not exclusively, being used to express sensations of pain. But if you say to your dentist, 'it hurts!' or if you say 'Stop!' and if your intention is that he stop drilling, then at least one requisite imperative condition has been satisfied.

However, this intention that an act be done is not alone sufficient to account for our speaking of such expressions as 'having meant something'. If you were to scream in the dentist's chair and if your only intention were that he stop drilling, you would not say of your scream, 'I meant by it "stop drilling!"'. For you may have intended the scream to cause him to stop drilling just as your leaping out of the chair suddenly would have caused him to stop. Such activity is not employed 'to mean' something. Or consider the diabolical man who wishes to do away with a certain troublesome woman. He commits a crime, sprinkles liberally around the scene of the crime the woman's exclusive perfume,

intending that the police arrest her, and intending that she be convicted of the crime. We should not say of his act that he meant by it 'arrest her!'. An additional intention of some sort is required.

Now, you may have intended your dentist to stop drilling and you may have intended that he recognize your intention. Here we have a scream used, it would appear, as a sign of an intention and resembling those signs which form a part of language systems. Can we then say of your scream 'You meant by it "Stop drilling!"'? Though a step forward, such a formulation is still inadequate. Consider these two examples, the first perhaps a little far-fetched. I may point to the door, catching your eye. I may intend that you recognize my intention. But suppose my aim is to make you leave because I have hypnotized you into doing so, into recognizing my intention and being unable to withstand it. I may have meant nothing by my pointing; it may have been used to draw your attention to my eyes. Consider, next, the case where a waiter in a restaurant, eager to leave for the evening and quite put out with the couple who seemingly will never allow the restaurant to close, comes over to their table continually, rattling dishes making it impossible for the couple to continue their conversation. Here he has the intention that they leave, the intention, no doubt, that they recognize his intention, but they are induced to leave by the rattling of the dishes, not necessarily in any way by a recognition of his intention that they leave. We would not say 'the rattling of the dishes meant "leave!"'. The rattling of the dishes simply induced them to leave. It is not enough that the appropriate intentions be temporal concomitants of the use of a certain expression. The recognition of intention must play its role in inducing the person to act.

When we issue an imperative we intend, not simply to induce a belief in the person addressed concerning our intentions, but also that his recognizing our intention induce him to act. If this were not so, an imperative use of language would reduce to the conveying of information to another person about our intention that he act plus an intention that he recognize that we have such an intention. But conveying such information and intending such recognition is not all that we intend when we issue an imperative.

We intend that the person addressed act and intend that the recognition of our intention induce action not just belief. We can now propose criteria for judging whether an expression is imperative or not.

X uses an expression imperatively, if and only if (1) X intends for Y (the person addressed) to do an act and (2) X intends that Y recognize X's intention that the act be done and (3) X intends that Y's recognition of X's intention induce Y to act. And now, whether in fact the person addressed recognizes the intention, affects that person's understanding of what the expression is being used for. It does not affect the characterization of the use to which one is putting language. Many children do not realize that they are being told to do things. The person addressing the child may nevertheless be speaking imperatively.

I have set out in detail and as convincingly as I can an analysis which initially seemed to me quite adequate and which might well convince others. It seems to me now that it must be rejected. I shall list several types of cases, several of which cause embarrassment for the foregoing treatment. a) When the actor on stage says, 'Lay on, Macduff!' or 'Get thee to a nunnery!', it would be queer to suppose that the intends that the actor near him actually lay on with a heavy sword or that he has a favourable disposition toward the lady near him on stage leaving for a convent. b) Suppose that after a long trial and a good deal of suffering you are 'on to' your son's ways. If you tell him to do one thing, he is quite sure to do the opposite. He is consistent in this respect but in few others. One evening, wanting very much for him to go to bed, you say, 'Look here, David, be a good boy and stay up und watch television!'. Off he scampers to bed. There has probably been many a movie in which the villainous sergeant who is 'out to get' the hero-private orders the private to do something, intending, not that he do it, but that he disobey, so that penalties can be imposed. c) There are those situations in which expressions in the imperative mood are used for purposes of humour or insult or to convey incredulity. I notice, for example, a friend coming out of the library, his head bowed, his gait quite slow. I know that he has been there every day for weeks, working at least

eights hours a day and I know, too, that he is in great need of a rest and I want him to have a rest. I say with a smile, 'Get back to work, lazy chap!'. If someone attempts a particularly bad pun, ha may hear, 'Oh, jump in the river!'. On occasions one may say, 'Go to the devil!' or 'Tell it to your Uncle Charlie!'. I suppose that 'Go to the devil!' is usually meant as a serious or humorous expression of anger. It does not purport to tell the person addressed to commence a journey to a notorious cloven-footed rebel. The second expression is one used to express complete incredulity. One uses it if one knows that the person addressed does not have an Uncle Charlie. If the person had an Uncle Charlie, it would lose its significance. One is saying, 'Bosh!' in a colloquial way and doing it in the imperative mood. d) Jesus may have said, 'Let him who is without sin cast the first stone!'. It is a sentence in the imperative mood. But Jesus did not intend for anyone to cast a stone. He intended, we may conjecture, to point a moral, to have it grasped by those present, and to have this restrain anyone's casting a stone.

One can handle such cases in either of two ways. One can simply hold to the analysis already presented and mention these contexts as 'odd' or one can admit the inadequacy of an analysis of language used to tell persons to do acts which requires an intention that they do the act they are told to do. The cases mentioned in (b) are the troublesome ones. In the other situations we describe what is going on as 'acting' or 'joking' or 'pointing a moral', but in (b) we describe it as an order and an adequate analysis of imperatives should be able to incorporate such cases rather than putting them aside with a remark such as 'insincere contexts don't fit the analysis.'

An officer may order a subordinate in the following way, 'you will appear . . .!' Now what makes this an order and not a prediction if the officer's aim is to induce disobedience as it might well be? We can't say that 'he intends that the subordinate act as ordered', for this is ruled out by hypothesis. The formulation which I think most adequate and which should replace the one earlier stated is as follows: the speaker intends that the person addressed recognize the speaker's gesture or utterance as one used

to tell a person to do an act. It is irrelevant that the speaker's purpose in saying what he does deviates from what is normally the purpose of persons saying what he does, and it is also irrelevant that the person addressed is aware that the purpose is of this kind.

Order Situations

Introductory remarks

What is implied when it is thought appropriate to say, 'he ordered me to . . .' ? For those primarily interested in the law, the verbs, 'to order' and 'to command' are of special interest. Several types of situations in which persons order and all command situations share at least one important notion, the notion of 'an obligation', with legal situations. And in defining 'Law', or offering a framework with which to approach the law, legal philosophers have been especially interested in 'a command' as the basic notion.

The analysis which follows is burdened by a number of difficulties which may lead to serious confusions if they are not first indicated. Ordinary usage has few clearly defined usages for the verbs 'to order' and 'to command' and the corresponding nouns 'order' and 'command'. Often, 'to order' and 'to command' are used interchangeably. If one says, 'I order you . . .', in many situations we could substitute for that expression 'I command you . . .'. The noun 'order', too, might be used in certain circumstances in which we should be reluctant to speak of anyone's having 'ordered' anyone else. The declaratory order of a court is one such example. There are also situations in which an order is 'made' or 'issued' and in which, applying our criteria for an imperative use of an expression, we could not speak of 'imperative expressions'. We shall have to analyse this sort of situation with some care because of its affinities with a situation in which a law is 'laid down'. The noun 'order' may be treated as synonymous with 'regulation'. There are important legal situations in which this is so. When we 'order' from a menu in a restaurant or 'order' an automobile from a dealer, an analysis of such orders in terms of requests seems

more appropriate and I shall not discuss them. One verb, the verb 'to order', is used in ordinary language to describe the use to which an expression has been put in situations which clearly differ in respects significant for lawyers and moral and legal philosophers. We have, therefore, 'to snuff with distinctions' and we shall introduce some usages so as to make explicit the differences of importance. This will be done by the simple device of attaching various adjectives to the word 'order' which it is thought will appropriately indicate the distinctive use made of the imperative expression. The word used is not of chief importance in the study that follows. Ordinary language gives a hint and we start with it, but we do not adopt the position that because the same word is used it 'stands for' the same activities. It is important to recognize these different situations. And ordinary language is of some help. There are characteristic situations, for example, in which we should feel uncomfortable using 'order' language and should think it 'felicitous' to use 'command' language and vice versa. There are principles upon which these situations may be distinguished.

The method used in the following pages needs a word, perhaps of justification for those who may wonder 'why all this attention to the language we use?' 'why all this bother over words?'. There are several reasons for this bother. First, it is no news that language is a tool which, if used without care and sensitivity, may lead to all sorts of confusions. Paying close attention to the language which is used to tell persons to do acts, to regulate conduct, to ascribe obligations, etc. may lessen the chance of confusions arising and may permit the dissolving of confusions which now exist. Second, the language we use is something conveniently at hand, at least for those of us who have learned it, which has captured distinctions and connections which people have found worthy to note. If we 'consult' it, these distinctions and connections come to our attention. To be sure, there may be good grounds for supplementing our language. We do so on several occasions in this work. To treat the language as sacrosanct may only prove confining, but to fail to commence inquiry with it may only prove presumptuous. Finally, if we look carefully at the language we use, brought to our attention may be inadequacies

and limitations in that language. By holding our language up before us for examination and apart from the world, we may better recognize its limitations and perhaps better grasp the world.

Let us continue now with the presentation. The phrase 'order situation' means in this study a situation in which an expression has been used imperatively, and it is appropriate to say 'the person used that expression to order ...'. Such situations will be divided into two classes: Personal orders and authoritative orders.

Let us now consider *Personal Order Situations*. The person ordering in such situations has no authority to do so. A person has authority when a group of rules exists which provides the person with a right to order.

Personal order situations are best illustrated by the armed thief who orders us to hand over our wallets and put up our hands or the adult who orders a child, not his own, to fetch him a newspaper, threatening harm in case the child does not comply. A linguistic peculiarity of these situations, one which can, I believe, be explained, is that one does not say, 'I order you to ...'. This linguistic form is reserved for situations in which the party ordering has authority to do so. In personal order situations, then, there is not present the notion of 'he had the right to order me'. The person has neither moral nor religious nor legal authority to which he can appeal in justifying his order.

When we say, 'the thief ordered the boy to keep quiet' we imply that the thief had an intention to coerce compliance when he issued his imperative. A person is employing the verb 'to order' in a sense different from the one here employed if he says, 'I ordered him to do it, but I had no intention to coerce him into doing it'. But this is not to say that an expression like 'keep quiet!', used as an order, is equivalent to a statement of this sort, 'I intend to harm you if you don't keep quiet'. An order is not a statement of one's intentions, though something about one's intentions is implied when an order is issued. It is not an order if I privately have not the appropriate intentions. The reception that the expression receives is not the criterion for its classification. The intention of the person issuing the imperative is the criterion. Someone may conclude that I have ordered you to do an act be-

cause my behaviour falls into a category of behaviour not unlike that of those who generally order. The determinative voice, however, is mine. The question 'are you asking me or telling me?' makes sense and implies a determination of classification by reference to the speaker's intentions. One may, of course, criticize those whose behavior has been misleading, for orders generally take a form that can be distinguished from requests.

Generally in such order situations the person who orders makes apparent some harm in his control when he issues the imperative and intends by so doing to coerce compliance. But the question put forward above – 'are you asking me or telling me?' – indicates that this need not always be so. A person may order without associating with his order a display of coercive powers. However, in such situations, if there is non-compliance with such an order, there is an intention to coerce compliance with it. When one orders there is always an intention to coerce compliance though the person ordering does not necessarily make apparent at the time of issuing the order his ability to coerce. The normal situation is one in which he does make his coercive abilities part of the situation, though, of course, in what he says he need not make direct reference to his power to harm. We shall in this study keep to the normal situation.

There need not, however, be ability to inflict harm in such situations. Suppose the thief waves his gun and says, 'keep quiet!' and suppose he intends inflicting us with harm if we make the slightest sound. If, in fact, the gun were empty and the thief did not know it, would it be 'a request'? An order is not a prediction of what will occur any more than it is a statement of fact. It is an order because of an intention related to coercion and one may be coerced by threats of harm.

Our remarks on intention to coerce and ability to do so in personal order situations raise a number of interesting problems. Suppose the person issuing the imperative knows that he cannot harm the person addressed because the bulge in his pocket is a stick and not a gun. If he says, 'hand over the money!' is this an order? And, too, how much coercion need he intend or represent? When borderline cases are imagined, difficulties of classification

are bound to arise. We can best proceed by illustrations and conjecture as to what would be an acceptable classification. Let us first take a rather exceptional set of cases. Suppose a child says to his parents – they are imagined in the bloom of life and quite able to control their child on the whole – ‘Leave the television set on!’. Now suppose the following three cases: a) The child intends bursting into tears if his parents do not leave the set on and makes his intention apparent to them. b) The child intends throwing a temper-tantrum which has been rather effective in the past when his parents have shown a reluctance to comply with his imperatives and he makes himself tense so that they are aware of what might be coming. c) The child has a revolver in his hand and intends using it in case of non-compliance. In which of these cases, if any, should we think it appropriate to say, ‘he ordered his parents to leave the television set on’?

First, our decision to classify one way rather than another will depend mainly on the number and importance of elements in each of these situations shared with what we vaguely, perhaps, regard as a standard or normal case of ordering someone to do an act. Such a case would be not unlike a thief ordering us to do an act and having a revolver in his hand, which he intends to use and which is loaded and in perfect condition. Second, in each of these cases there is a certain oddity in speaking of ‘ordering’. Children do not generally, if ever, order their parents to do things; they request or ask permission. We might wish to conclude that any such cases imagined are bound to be so odd as to make it impossible to classify them. Third, if the child intends merely to cry, our reluctance to treat it as an order is perhaps greatest. The sanction is so unlike the ones usually introduced in order situations, in both type and coercive value, that it would sound strange to speak of his sentence as ‘an order’. If it were so considered, because of the slight coercive value of tears, any distinction between requests and orders would be difficult to draw. We think this is a situation which should likely culminate in a spanking and not in compliance, and such situations are not order situations. Fourth, the temper-tantrum is also an odd sort of coercive device and the appropriate language of classification is difficult to

come by. However, we might be more willing to treat this as an order because in these imagined situations it is an unusually effective, though strange, means of gaining compliance. The parents are expected to be accustomed to tears; their genuine fear of a temper-tantrum is more understandable. Fifth, an obviously effective and customary device like a revolver and the implied destruction of the familial relationships in the willingness to use it, enables us to classify the case as an order without too much difficulty. 'Imagine that! He ordered his parents about with a revolver! Have you ever heard of such a thing? What a child!'

Now let us move away from this extremely odd type situation. The examples seem to indicate a greater willingness to place within the class of orders those expressions used in situations in which the coercive device is reasonably effective. Now suppose a thief orders you to put up your hands and turn with your face to the wall. Suppose he indicates to you that he has a gun by a gesture of his hand in his pocket. Suppose that in fact he has no gun. Once again we are confronted with the odd situation. To call it a 'request' seems strange. It is much more like an order. Perhaps we should think it best to say, 'he demanded . . .' but 'order' and 'demand' seem alike in what they imply. What should we say when we discovered that he had no gun and that we had been fooled? 'He told us to put up our hands and turn our faces to the wall.' 'But did he order you?' 'Well, that all depends on what you mean by "order".' In this situation I think we should say that 'he ordered you to put up your hands' or 'he ordered me to put up my hands but he was faking all the time about the gun'. Knowledge of an inability to harm on the part of the person issuing the order does not affect the classification of the utterance. This is only, then, to repeat that there must be an intention to coerce. One may be coerced by fear resting on an erroneous belief.

Second, suppose the thief's power lies only in his fists and you are a champion boxer, but the thief is unaware of this. Such a case seems to fit nicely into the class of 'orders', for the thief intends to exercise what would be coercion of a reasonably effective kind in most situations. Third, suppose the thief and you are

both aware of each other's knowledge that what he intends doing in case of non-compliance will be of no effect. The situation imagined is ridiculous and is not classifiable.

Assume finally that the criterion we established were 'the slightest chance of incurring the slightest evil'. This suggestion made by John Austin in his *Lectures on Jurisprudence* is inadequate for several reasons. a) If the person believes he can harm, but if in fact his belief is erroneous, this does not affect the classification of the expression as 'an order'. He may believe the gun he is holding is loaded and in fact it may have been secretly emptied. b) If the person knows there is not the slightest chance of the slightest evil he may still pretend there is and his expression would be an order. It is the coercion that matters. c) 'The slightest evil' is a poor criterion, for in most order situations the evil intended is reasonably expected by the person ordering to effect compliance. Suppose our thief is a keen student of human nature and suppose he intends breaking down and weeping if you do not hand over to him your wallet. And suppose that his belief that you will be terribly embarrassed is rather well-founded. There is, then, 'the slightest chance of the slightest evil', but it is the sort of situation one would be reluctant labelling 'an order situation'. d) The criterion suggested would destroy the distinction between requesting and ordering, for often a request not granted is followed by 'a long face' and this may have some harmful effect on the person denying the request. A conclusion which seems warranted from these illustrations is this: Excluding the exceptional cases where coercion is not made apparent when the order is issued, we may say that the use of the word 'order' is regarded as proper under the following conditions: a) The person has an intention to coerce compliance and b) The person ordering reasonably believes that the coercion represented will cause genuine harm to the person addressed so as to coerce the choice of that person by making less eligible to that person a choice of conduct.

It would seem to follow that the harm which is threatened must be in the control of the person ordering. This inference requires some discussion of the relation between orders and warnings. Certain warnings would not be considered orders. The whisper

'don't move!' as the rattlesnake is about to strike is a warning and not an order: a) with an order the harm which is threatened is in the speaker's control; b) the purpose of the person ordering is to satisfy his own interest, not the interest of the person ordered. But what of those situations in which we clearly are in another's control? Suppose we have been kidnapped by a gangster, and he shouts at us, waving his gun 'don't move!'. We might say, describing such a situation, 'he warned us not to move'. However, such situations might more appropriately be described by saying, 'he ordered us not to move and threatened us with being shot if we did'. Warnings, then, are still related to a harm not in the speaker's control and the word 'threat' is substituted. However, consider the same gangster situation and suppose he says, 'I warn you two, don't make a move!'. 'He warned us' is appropriate and the harm which may result is in his control. We can conclude, I think, that usage is unsettled here. We do speak of warnings when the sanction is in the control of the speaker. But warnings, unlike orders, seem more directly to emphasize the harm that may result in case of non-compliance, are issued in consideration of another's interests, and can apply to natural as well as other types of sanctions. When we warn we issue an imperative in consideration of another person's interests. With an order, we intend for the person to act and we intend for our words, when understood, to induce certain behavior and we intend to support this intention by coercion. We order in consideration of our own, not the other person's interests.

Usage here, however, is once again not fixed. We often speak of doctor's orders where the harm which will result is not in his control, but is the consequence of disregarding his orders. The classification of this sort of case is obviously difficult; it resembles in some respects 'order', but in other respects it resembles 'advice'. I think the classification which is most appropriate – if one need classify such cases at all – is as advice but advice made urgent because of the other factors in the situation. Intentional coercion plays no role in the doctor situation. Besides the possibility of serious consequences if the imperative is disregarded – a strong resemblance to orders – we have a vague notion that 'what

the doctor says, goes'. This puts the doctor in a position almost of authority over us. 'I'm in your hands; whatever you say, I'll do.' These two factors seem to account for our using the word 'order'.

Personal orders are characterized by 'urgency'. When we are ordered to do an act, we do not think, 'I can do this or not; it is as I please'. We think, 'he really means for me to do it and if I don't, things will be terribly uncomfortable for me'. But this is not a defining characteristic of orders of this sort. A person may order me to hand over my wallet and I may know that the revolver he has poised for action is really empty.

There are at least three considerations which have led to a classification of such orders in this study as 'personal'. First, the imperative is issued with the personal interests of the person ordering as predominant. This is why one troubles to order. The interests of those ordered may be affected, but one is not motivated to order through a concern for their interest. On the other hand, with directions, advice, and warnings, the interests of the person addressed are primary. The imperative is issued ostensibly to satisfy the interests of the person addressed. While in the case of personal orders, 'because I said so!' is an expected reply to the question 'why should I act as you have told me?' in the case of advice it is not.

Second, personal orders are usually given in face to face situations, from a person ordering to a person addressed. They are seldom written, though here too there are exceptions. The kidnapper may order the sorrowing parents by means of a letter to leave the ransom money in a particular place. Personal orders, too, usually relate to specific occasions and are not general in formulation. But there are exceptions here as well. Our kidnapper may order us to 'put £500 in an envelope each Tuesday for a period of two months and post it to . . .'. The racketeer who gets his monthly 'donation' over the years may have ordered the complying shopkeeper. In such cases one has ordered a person to follow a rule. A rule, however, which is laid down, in formulation is general both as to persons and acts. An order is not. An order specifies a person and usually specifies the occasion upon which

an act is to be done though it need not do this. The standard order situation is one in which the order is addressed orally to a specified person to do certain specified acts.

Third, the person who orders does not have authority to do so. Consequently, he is not ordering in his capacity as an officer whose office is defined by rules nor is his status one of moral or legal authority such as a parent's. It is John Smith ordering and not a policeman or a judge or a club president or a parent in their respective authoritative capacities.

The order, in these cases, can be said to oblige the person addressed to act in a certain manner. However, principally because of the absence of authority in the person ordering, this 'obliging' is of a particular kind, one differing in important respects from the obligation which arises in other cases of authoritative orders and commands. We have now to examine this notion of 'obliged to act' as it is present in such situations.

The concept 'coercion' has entered the discussion at crucial points. It is necessary now to look at it more carefully. When a thief orders you to do some act and threatens you with his gun, what is implied by the sentence, 'you were obliged to act as he ordered' or 'he coerced you into acting'? This is the question to which we want now to address ourselves. First, we should not say, 'you are under a duty to him to act as he says' or 'the order was binding' or 'you have an obligation to him to act as he says'. This is clear. We think such language is appropriate only when the demand made is a 'legitimate one'. And an order or command is legitimate if the person has a right to order or command and exercises that right in the situation either in accord with rules or as a practice permits. The right depends on certain rules which have been laid down and which are on the whole efficacious, that is, on the whole obeyed and if not obeyed, on the whole enforced, or it depends on rules or practices which, though not laid down, are generally accepted. In a personal order situation there are no such notions. We may be 'obliged to act' or 'coerced to act' but there is never the notion of 'having an obligation to the person ordering' to act as he has ordered. Second, we cannot be coerced into doing the impossible. No one can be coerced into jumping a

hundred feet into the air. Nor do we speak of a person being coerced when he is simply moving in accord with the laws of nature. The wind which lifts us up and carries us is not coercing us. We do not speak of 'human action' in such cases; when a person is coerced he acts. Third, there are situations, perhaps, closely resembling the foregoing ones, in which physical force is directly applied to a person compelling him to do something. For example, 'his shoving a gun in my stomach compelled me to cry out in pain'; or 'he compelled me to leave by pushing me out of the door'. In such cases we normally should not talk of being 'coerced into' a line of action. Fourth, the above illustrations lead to the conclusion that when one is coerced to do an act, one has a choice to do or not to do the act. In some sense 'being obliged' or 'being coerced' implies a loss of freedom of choice, but it is not the case that we have lost freedom to the extent that we say 'it is impossible to do otherwise'. To be obliged or coerced is to have had some limitation placed on one's freedom of choice, but it is still to have before one a choice. Fifth, connected with the foregoing point is the fact that when we are coerced into action we do the coerced act intending to do it even though we do it against our will. You do not coerce me into shooting John if you should happen to push him in front of me just as I am firing a gun at some distant target. I shoot him accidentally and I am not appropriately said to be coerced. Nor should I be coerced into poisoning Jane if, without my knowledge, you substituted arsenic for sugar and I, courteously, provide her with arsenic for her coffee. I am not coerced into the act if I do it by mistake. Sixth, the person who is coerced believes that the consequences of his omitting to act in a certain way are worse than if he acts in that way. If, whether or not the person acted as ordered, he knew the harmful consequence would be forthcoming, he would not be obliged or coerced into acting as ordered. The loss of freedom involved in being coerced relates to the consequences which are harmful and which one is threatened with in case of non-compliance. It is the imposing upon one person (the analysis is limited to inter-personal situations) by another person of a more 'obstacle-ridden' course for a less obstacle-ridden one. It is fear of this more obstacle-

ridden course which makes less eligible to us that course and which induces action which we say is 'coerced action'. In summary, then, X coerces Y if these conditions are satisfied: a) X interferes with Y's choice by making less eligible to Y a course of conduct; b) X's interference leads Y to do an act he is unwilling to do; c) Y is induced to act because of fear of painful consequences.

Let us now consider *Authoritative Order Situations*. Authoritative order situations are ones in which a person orders another to do an act and in which the person has authority to order, but in which the authoritative order is neither issued as the end product of a process, the procedure of which is governed by rules (such as a 'cease and desist order' of a court or administrative agency) nor is itself an established part of that rule procedure (such as the standard type of subpoena). Such orders are, even though divorced from rules in the ways indicated, legitimate or proper only because rules or a practice provide the person ordering with authority to issue orders under certain circumstances and sometimes in certain ways. Because of the nature of the occasions for their use, we may speak of these orders as 'incidental' authoritative orders, but 'incidental' should not be understood to mean that they are orders which have no important consequences.

These situations are illustrated by the following exemplification: a policeman ordering a thief to stop or a mob to disperse, a judge ordering a defendant to keep quiet or a club president ordering a member to cease disturbing the meeting. In all such orders it seems appropriate to say, 'I order you to . . .'. In such cases there is an obligation to act in the way we have been ordered to act.

What is implied when we say that someone has such an obligation? First, when an order is issued in such situations we do speak of a resulting 'duty to obey' or 'an obligation to comply with the order'. However we should not use the phrase 'a binding order'. That phrase, though closely related to the use of 'an obligation to obey', does not find in these situations appropriate conditions for its use. 'The order is a binding one' indicates that one is 'under an obligation' to do an act at some future time specified

or not. In these situations, however, the act is to be done immediately and the judgment is therefore inappropriate. It is applicable to formal orders (a class soon to be analysed), some commands, and rules. Second, the person who orders intends to circumscribe the behaviour of the person addressed. Third, the obligation does not necessarily depend on one's being coerced. If one issues an order and coerces by threats of harm this may give rise to a duty of resisting rather than of compliance. Thus, the villain ordering us to kill a helpless child may oblige us in the sense of 'oblige' discussed under personal orders but we are, because of the existence of certain rules, under both a moral and a legal obligation to disobey him. Further, the absence of opportunity to enforce the order does not affect the duty to obey. The thief ordered to stop by the helpless policeman is under no less an obligation to stop because he is certain to get away. The obligation in such cases is not a prediction about the possibility of enforcing the order. The obligation must be something apart from the ability to enforce the order, for we speak of 'enforcing obligations' in these situations and imply they exist prior to enforcement. Fourth, the feeling one may have of an obligation to act as one has been ordered to act cannot be the criterion either. The obligation exists in cases where one does not have such feelings. The thief, for example, may not have such feelings and yet is under an obligation to obey the policeman. And one may 'feel' that one has an obligation to obey and yet not have such an obligation. Fifth, where an obligation exists in such situations, the person who has ordered had authority to order as he did. If the authority is absent or somehow misused (in ways we shall soon investigate) the obligation does not exist or is seriously affected in ways determined by rules. If power to coerce obedience or enforce one's order is absent, however, the obligation is treated as unaffected. Sixth, we can now make explicit the following conditions for the existence of an obligation in these cases: Y has a duty to obey the order of X, or is under obligation to obey the order of X, if and only of a) there are in existence rules or practices. The rules may be of a moral or religious or legal or quasi-legal kind, etc., b) the rules or practice provides X with authority

to issue orders, which authority is generally conferred in terms of 'rights' or 'powers'. For example, 'the umpire shall have the right to order a player from the field', c) X has ordered Y to do an act, d) X has ordered Y to do the act in conformity with rules prescribing the content and manner of ordering. But for the class we are now considering, the order has neither been issued or 'made' as an end product of a process the procedure of which (for example, a judicial hearing) has been governed by rules nor is it an integral part of that rule procedure.

What role is played by coercion in authoritative order situations? In fact, as a practical matter, the person ordering is in legal and quasi-legal contexts usually provided with the power to invoke sanctions under rules of the system in case of disobedience. In most cases, however, obedience occurs, not because of fear of a sanction – a marked difference from personal orders – but because of habit and respect for authority. We might initially think that such orders may be issued without a person's intending to coerce compliance, but this does not appear to be so. Consider the following situation: A crowd of people has formed near an automobile accident and a police-officer moves among them saying, 'move along there!'. We move away and do so respecting his authority. We should say that 'he asked us to move along and we did, complying with his request'. Suppose now we notice a stubborn man who has disregarded the officer's request. The officer repeats, 'move along, sir!'. The man is still too curious to do so. And now the officer says, 'I order you to move from here!'. The situation changes and becomes an order one. Appeal is made to one's authority to order and there is the inference to be drawn that sanctions will follow if there is non-compliance.

Consider now a more troublesome sort of case. A police-officer orders an armed bandit to halt. The policeman is not armed himself, has no opportunity to compel obedience practically speaking, and the bandit is about to enter a plane which is to take him to another country which does not extradite. Whether, in fact, one can enforce one's order is a question about its efficacy in getting the bandit to stop, not about the nature of the expression and what follows from its utterance in terms of obligations. If the

bandit 'gets away with it', we say 'he ought not to have done that', recognizing that effective coercion is not a necessary condition for an order's having been issued or for its being obligatory. But in such order situations is there implied an intention to coerce compliance? The police-officer is seeking to induce behaviour by appeal to the respect for authority that persons normally have, but is there coercion? Normally, the appropriate sanctions are present, and while in this atypical situation they are not, none the less because normally they are, we may still speak of the officer 'ordering'.

Authoritative orders are less personal than those of the first group of orders. When a police-officer or judge orders, his particular interests are not predominant. He orders generally because the rules governing the office he holds require it. The ordering occurs usually because the officer appreciates the need to satisfy interests other than his own. An umpire may order a player off the field because the player has shown disrespect for the rules. But the umpire may personally have wanted to display the same disrespect and is precluded from doing so because of the position he holds. Second, the orders of an authoritative person are orders issued by 'an office' or 'officer'. For example, 'the judge' or 'club-president' orders. Third, such orders usually do not oblige individuals to follow a course of conduct. In most cases they are not used to order the adoption of a rule of conduct. No generalization here, however, could be safe from exception. Consider the judge ordering counsel, early in the trial, not to interrupt opposing counsel during cross-examination. Like personal orders they are generally given in face to face situations. Because of their close connection with the specific situation in respect of its minor and momentary changes, such orders are not usually written.

What types of judgments are appropriately made about such orders? We judge on the basis of 'internal' and 'external' standards. These standards are rules regulating the behaviour of officials or other persons in authoritative positions in their issuance of orders. When we say, 'he didn't have the right to order . . .' or 'I am under no obligation to obey such an order . . .', we may be judging the person's behaviour who issued the order by conformity

of that behaviour to the standards provided by the system, which has defined the functions of the office which is issuing the order. For example, a judge is operating within a legal system which defines his office. We may be judging by standards outside the system. For example, a moral system may be appealed to in criticism of an act within a legal system. When one says, then, that 'the judge did not have the right to order me . . .', this may mean that the judge's order in some way violated the rules obliging the judge to act in a particular way that are peculiarly legal rules, or that though legally proper, the behaviour in issuing that order was morally wrong, or that it was both morally and legally improper.

We may challenge the existence of an obligation under the following conditions. First, the person may have ordered assuming he had authority and we say, 'he did not have the right . . .' meaning by this that he really did not have the authority. There is simply absence of authority to order. If the club secretary tells me to keep quiet or to sit down and stop disturbing the meeting, I may respond, 'your job is to keep the minutes of the proceedings. Only the president has the right to order me to sit down and stop speaking'. Second, there is criticism where the person exceeded the proper limits of his authority. The judge says to counsel during his concluding argument, 'keep your hands at your side and don't wave them about as you have been doing'. And the counsel replies, 'you have no right to tell me to do that'. Third, there is criticism based on abuse of authority. Here there is an offence because one has exercised authority in a manner which is prohibited by rules regulating one's office. A police-officer, for example, who has ordered you out of the way of an official car by pushing you with his stick might be criticized on such grounds. If there is absence, excess, or abuse of authority, there is no duty to obey. Condition (d) laid down for 'an obligation' has not been met.

Let us now consider *Formal Orders*. The word 'order' is employed to characterize the use of imperative expressions in situations differing in various important respects. There is another important occasion for the use of the word with which we must

now concern ourselves. I wish to discuss at this point several situations which, though resembling in some respects those order situations already discussed, differ fundamentally from these.

Let us call 'a formal order situation' a situation in which an order is issued by persons in authority or by an office and in which the order in both content and manner of issuing is closely limited by rules. Unlike 'authoritative orders' the orders in this class are not incidental and related to the momentary aspects of a situation. But the line I am proposing to draw here must not be taken as a perfectly straight one. Numerous cases would arise where the difficulty of classification would be great. These orders impose obligations on persons and can be said to be 'binding'. The situations I have in mind are illustrated by the following: the court issues a subpoena ordering a specified person to appear in court on a certain date; the court issues a writ of mandamus ordering a public officer to provide certain parties with opportunity to be heard in an administrative hearing; an administrative agency orders the deportation of an alien because he has falsified his records; the court, after a trial, orders that defendants 'cease and desist' from burning sulphur in their back yard.

The personal element in such situations is generally unimportant. It is not a situation in which one could appropriately answer the question, 'why should I obey?' by saying, 'because I personally wish you to'. Action proceeds according to rules, not personal wishes. The face to face situation is a rare occurrence with most orders of this type. The formal order is generally written and the form is prescribed by rules. These orders are still directed to specified persons (there are some minor exceptions where the ordered party is a class of persons). They usually relate to specific situations and do not prescribe a course of conduct though a cease and desist order may be considered a notable exception. Non-compliance is the occasion for the application of other rules which provide officials with authority to invoke certain sanctions. The situations, then, are primarily legal in character. There is no implication of a personal intention to coerce similar to the intention that was implied in case of other orders.

But the chief distinction between formal orders and orders of

our other two classes is this: formal orders are not imperative expressions. An order is 'made' or issued from an office or institution directing that a specified person do an act, the order is binding, but it is 'an order' whether or not anyone has an intention that the issuance be recognized as requiring an act. But how can this be so?

In existence is a legal system and applicable to the situation are a complex of rules, some of which we shall now indicate though a far more detailed analysis of rules is clearly required. a) There are rules which provide that 'the court' may, under certain conditions, order an individual to appear as a witness in a proceeding. b) There are rules which prescribe when and in what manner this order may be issued. These rules provide standards which must be met if the order is to be considered valid. And if it is not valid, this has rule-determined consequences, for example, the impropriety of imposing fines for non-compliance with the order. c) There are rules which provide that the person to whom an order is directed must obey the order. These rules give to the pronouncement – even if it is expressed in the future indicative ('Mr. Jones will appear . . .') – the status of an official order which directs a person to do an act and which, consequently, may be 'satisfied' or not. For these rules, by providing that the pronouncement must be obeyed, establish the pronouncement as an order – something which can be either obeyed or not. If the pronouncement were a prediction, this notion of disobeying – of not satisfying the order – would be inappropriate. d) There are often rules which provide that if there is non-compliance with the order, a certain penalty is to follow. These rules support the rules of (c). e) There are rules which provide what the penalties are to be for non-compliance.

The rules, then, in such situations are substitutes for the imperative intention. Certain types of 'orders' need not rest on intentions. Let us consider two cases which will bring out the irrelevance of personal intentions, in such formal order situations.

First let us take a formal order which is an intimate part of a rule procedure, a subpoena issued by a court, ordering a witness to appear at a specified place and time. Suppose the defence at-

torney provides the clerk of the court with a list of names and addresses of parties whom the attorney wishes to appear as witnesses. The clerk proceeds then to place the court seal on a prescribed form, fills in the names of the parties, and another court official delivers the subpoenas as prepared. The clerk goes about his task in a mechanical way. Personally, he is unacquainted with the parties. It is of no interest to him who appears in court as a witness in a trial about which he knows nothing. He just does the job which he is paid to do. The subpoena is an order of the court. It is irrelevant whether or not he forms the intention that the subpoena be treated in a certain way. A person, then, is 'ordered' to do an act; an obligation is created to do that act; and it is not necessary that anyone have the appropriate imperative intention.

Let us next consider a rather elaborate example which will point a moral of great relevance for the theory that law is a command. Suppose the Court in our district is composed of five judges. Their names are Aaron, Beetle, Clover, Dandy and Elk. There are certain rules of procedure concerning the disposition of cases arising before them which these judges follow. One such rule is that if in any case before the Court a majority of the judges decide in a particular way, the decision of the majority is to be considered the decision of the Court. They also have adopted a procedural rule which provides that each of the judges is to deliver to the Chief Judge on a piece of paper his decision. The decision might read quite simply, 'I hold for the plaintiff. Beetle', or 'the injunction should be issued. Aaron'.

Now suppose before this Court, all judges present, the question whether Mrs. Sullivan, amateur scientist, may continue her experiments with sulphur in her back yard. The Court is entertaining a suit by her neighbours who seek to enjoin Mrs. Sullivan from her scientific pursuits, and they allege various types of harm to their interests. We are concerned with 'intention', so let us proceed to consider how the Court's final decision is reached.

Judge Aaron finds that the smell of sulphur is unbearable and he sympathizes with plaintiffs. He decides for an order prohibiting the experiments. He personally has an intention that she stop such experiments. Judge Beetle provides a more interesting ex-

ample and his reasoning is somewhat involved and perhaps exceptional for a judge. He personally intends that Mrs. Sullivan do as she wishes. She has appealed to him as a serious minded scientist, working on an important problem, and the defence she has made out has seemed to him strong. However, an earlier discussion with Judges Clover, Dandy and Elk has convinced Beetle that these fellow judges will decide against the prohibitory order. Beetle is not oblivious to court politics. He reasons somewhat as follows: 'The disposition of the next case is very important to me. I need Aaron's support on that case. If I decide with him on this one, I am sure that this will sway him in my favour on the next one. As far as the desirable disposition of the *Sullivan* case, Clover, Dandy and Elk will attend to that'. He decides for the prohibitory order. Unfortunately, Clover reasons remarkably like Beetle. He does not intend that Mrs. Sullivan stop her experiments. He, too, thinks that she should experiment with sulphur as she wishes and, indeed wishes that she would do so because of his dislike for the plaintiffs. Court politics, the desire to gain Aaron's favour, lead him, however, to support a prohibitory order. Dandy decides against the order as does Elk. But the Court has decided three against two for the order and it is issued. The Court orders Mrs. Sullivan to cease and desist from burning sulphur in her back yard. If she continues to burn sulphur, proceedings relating to the imposition of sanctions will result.

We have, then, an order of the Court. On no interpretation of the facts as supposed would it be plausible speak of 'intentions' that the defendant do an act. If our original analysis of imperative expressions were an adequate one, the argument could rest at this point, for it would only be required to establish that there was no intention that a person do an act and the order was still valid. It is necessary, however, to go one step further and establish the irrelevance for the validity of that order of intentions that the issuance from that court be recognized as an order. It is essential to note that such intentions are equally irrelevant to the binding nature of the court order. What establishes the injunction as an order is a compliance with a procedure such that rules provide for treating as binding an order which issues from that procedure.

The so-called 'will theory of law' or 'imperative theory of law' has much to commend it, for it brings to our attention features of legal systems which are similar to features of directive uses of language. I have tried in this study, however, to suggest one reason why such theories may mislead people. They will do so because an intention of a certain kind is implied in the imperative use of language and that intention is not, in fact, an essential constituent, though it may indeed be present, in legal systems which are rule-governed.

Existe-t-il un système de morale qui mérite le nom de système?

par

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Quand on dit «système» en parlant d'un système de morale, on devrait mettre ce mot entre guillemets pour montrer qu'on l'emploie dans un sens beaucoup moins rigoureux que celui qui intervient là, où par ex. on parle de système de logique. En effet, un système de morale, ce n'est ni un ensemble bien organisé, où chaque proposition a sa place par rapport aux axiomes acceptés, ni un tout cohérent. Parfois il ne doit son unité qu'à une tendance très générale à laquelle il emprunte son nom. C'est le cas lorsqu'il s'agit de morale dite d'honneur ou de morale appelée morale de renoncement. Parfois son unité n'est garantie que par le nom de son auteur. Bien qu'au cours des siècles la question de la méthode qu'il faut suivre dans les considérations morales ait été beaucoup discutée, on n'est pas encore d'accord, s'il faut construire un système moral *more geometrico* ou *more inductivo*, et ces deux orientations ont toujours leurs partisans.

Dans ma conférence, je passerai en revue quelques systèmes de morale, les uns procédant plutôt par voie de déduction, les autres se déclarant basés complètement sur les faits. Je les ai choisis parmi ceux qui ont une place honorable dans l'histoire de la pensée morale et constituent des modèles souvent imités. Bien que pour chacun d'eux je puisse me référer à de nombreux auteurs, je ne tiendrai compte que de ceux qui avaient l'ambition de créer un tout ordonné et manifestaient un goût visible pour un travail de systématisation.

I. Systèmes à tendances déductives

En tant que premier exemple, nous pouvons examiner les systèmes qui, en fixant au moraliste un but défini: notamment

celui de rendre les hommes heureux, croient déduire de ce principe toutes les règles de conduite. Ces dernières ne se borneraient qu'à recommander les moyens nécessaires ou suffisants pour atteindre ce but. Les relations causales entre moyens et fin seraient évidemment établies en ayant recours aux faits.

1. Je crois que la morale que Bentham nous propose dans sa «Déontologie» constitue le modèle classique de ce genre de système. C'est le seul penseur que je connaisse, qui développe ce système en détail, allant toujours jusqu'au bout de sa pensée, sans se soucier, si elle heurte les émotions, les préjugés et les coutumes.

Traçons en quelques lignes la voie que suit notre auteur. Selon lui, le but, que le moraliste veut atteindre, c'est le plus grand bonheur du plus grand nombre et il considère vertueuse toute action qui, pour se servir de son expression, «maximise les plaisirs et minimise les peines». On sait que Bentham évite l'erreur commise par J. S. Mill et qu'il ne fait aucune distinction de qualité, lorsqu'il s'agit des plaisirs. De leur valeur décide uniquement: leur proximité, leur pureté, leur intensité, leur durée et leur fécondité, tandis que l'objet qui les provoque est sans importance. Le seul principe définissant la répartition des plaisirs et des peines nous recommande de traiter chacun de la même façon. Bentham s'oppose à toute morale qui recommande le sacrifice et considère que ce serait faire preuve de folie que de renoncer à un grand plaisir pour faire un plaisir moins grand à notre prochain. Si cela fait plaisir à quelqu'un de faire souffrir les autres, le moraliste n'aura le droit d'intervenir que lorsque cette souffrance sera plus grande que le plaisir de l'agent. Il pourra alors rétablir les proportions en ayant recours à des peines. Le plaisir de l'agent puni sera désormais moins grand que la souffrance de sa victime, et il se peut même qu'il souffre au lieu de se réjouir. Dans ce dernier cas, le fait que tous les gens visent le plaisir et évitent la douleur – thèse acceptée par Bentham – incitera l'agent à changer d'attitude.

Comme toute critique, celle du système de Bentham était tantôt transcendante, faite du dehors, c'est-à-dire du point de vue d'un autre système, tantôt immanente, acceptant le point

de vue adopté par l'auteur. Ce n'est que cette dernière qui nous intéresse ici puisque nous nous occupons de la structure formelle des systèmes discutés.

a) Quand nous blâmons ou louons un acte, Bentham ne nous permet pas de prendre en considération l'état de conscience de l'agent. Ses motifs, ses intentions, ses efforts n'entrent pas en jeu et ne comptent que le plaisir provoqué ou la douleur évitée. S'il en était ainsi, les contours de la morale se perdraient complètement. L'infirmière qui, à l'hôpital fait des injections de morphine aux malades, aurait un grand mérite, si l'on tenait compte de la souffrance qu'elle soulage. La vache qui constitue la base alimentaire d'une famille paysanne serait également digne de respect vu son utilité qui, en fin de compte se traduirait par des plaisirs ou par des peines évitées.

b) Souvent, et non sans raison, on a reproché à Bentham de baser son système sur un calcul des plaisirs et des peines, comme s'il disposait d'une mesure qui lui permettrait de faire les comparaisons. Comment, en effet, décider, si la satisfaction de fumer une bonne cigarette est plus grande que celle d'admirer un tableau de Rembrandt, si nous souffrons davantage d'un mal de dents ou du choc que nous cause la nouvelle qu'un ami, que nous croyons dévoué, vient de dire du mal de nous en notre absence, si le plaisir que nous cause une conversation dans une atmosphère empreinte de cordialité est plus grand que le plaisir d'être loué pour un travail bien exécuté. La comparaison des plaisirs et des peines nous paraissant douteuse, même dans les cas, où nous ne tenons compte que de nos valeurs personnelles, pouvons-nous considérer un langage intersubjectif comme étant possible dans cette matière?

Ce calcul, pour les raisons que nous venons de mentionner, peut susciter des doutes, non seulement chez les théoriciens, mais aussi chez les pédagogues car, mis en pratique, il pourrait être dangereux. En profitant du fait que ce calcul n'a aucune base objective, l'agent peut toujours le résoudre d'une façon qui lui est favorable. Les textes de Bentham peuvent en fournir des exemples. Cet auteur est un des premiers moralistes, liés à la tradition chrétienne, qui tienne compte non seulement des hom-

mes, mais aussi des animaux. En effet, les animaux étant des êtres capables d'éprouver des plaisirs et des douleurs, on ne peut les oublier dans les calculs. Toutefois, Bentham n'hésite pas à affirmer que la souffrance des bestiaux tués dans les abattoirs est moindre que le plaisir de ceux qui se nourrissent de leur viande.

Tout en me rendant compte de ces difficultés je tiens à défendre Bentham contre ceux qui lui reprochaient de faire de la morale une comptabilité digne d'un petit commerçant. Bentham, dans sa vie personnelle, ne savait jamais bien compter, et s'il faisait de la morale un objet de calcul, c'était pour lui donner un aspect scientifique et pour la faire dépendre non des émotions changeantes, mais d'une arithmétique froide et précise. Le contemporain français de Bentham, Volney, était guidé dans son «Catéchisme du citoyen» par les mêmes ambitions.

c) Les critiques de Bentham ont souvent objecté qu'on pouvait obtenir la même somme globale de bonheur aussi bien en distribuant de petites portions à beaucoup de monde qu'en accordant de grandes à un nombre restreint de personnes privilégiées. En effet, il en serait ainsi, si Bentham n'adoptait que le principe de la maximisation du plaisir et de la minimisation des souffrances. Mais nous trouvons dans son système un autre principe, non dérivé du précédent, principe qui nous recommande de traiter chaque homme comme une seule unité et de ne traiter personne comme plus d'une unité (each to be counted for one and no one for more than one). Ce principe, clair au premier abord, peut être interprété de différentes façons. On peut le traiter en tant que principe de sélection nous démontrant, qu'il est complètement indifférent, qui nous choisirons pour diminuer ses souffrances ou pour augmenter son plaisir. On peut aussi le traiter (et cette interprétation paraît plus probable) en tant que principe de justice égalitaire qui s'oppose à tout privilège. Dans les deux cas, ce principe introduit dans le système un accent nouveau et joue un rôle analogue à celui d'une règle appartenant au méta-système dans un système de logique.

On sait que Bentham avait évité la faute commise par tous ceux qui, en admettant que la bonheur est le but ultime, finissaient par parler de vrai bonheur, notion qui, comme l'a si bien

démontré le prof. Dupréel, engage toujours un idéal de la personne. Mais ne distinguant pas le vrai bonheur d'avec le bonheur tout court, notre auteur énonçait dans ses jugements de valeurs un idéal de la personne sous-entendu. Il en était ainsi quand il parlait par ex. de flatterie déshonorante, ou quand il associait d'une façon inattendue l'effort à la vertu. Ces idées étaient dérivées d'un autre courant de la pensée morale. En portant atteinte à l'unité du système, elles montraient, combien il est difficile de dissocier une hygiène de la vie sociale de la pensée perfectionniste.

2. Au cours du XIX^e siècle, l'industrialization croissante suggère – comme on le sait – à divers penseurs l'idée de comparer la société à une usine où incombe au moraliste la tâche de veiller à son fonctionnement harmonieux et où les règles morales sont censées réduire toute friction à un minimum. Bien qu'il soit possible de ne voir dans ces systèmes qu'un cas particulier des systèmes précédents, celui qui vise à écarter les conflits, ayant probablement le bonheur de tous en vue, ces systèmes ont cependant une physiognomie qui leur est propre et méritent d'être examinés séparément.

Cette conception de la morale fut déjà celle de Hobbes. On se rappelle que, selon cet auteur, tous les hommes, pourvu qu'ils réfléchissent tranquillement, sont d'accord pour vouloir la paix. Une action est bonne, si elle contribue à la paix et elle est mauvaise, si elle entraîne un conflit. Du moment que la morale organise la vie sociale, il ne peut y avoir de morale qui ne traite les hommes qu'en tant qu'individus. Ce n'est que quand nous les considérons en tant que citoyens que nous disposons d'un critère qui nous permet de définir le vice et la vertu.

On se rappelle qu'après avoir approuvé cette tendance générale vers la paix, Hobbes en déduit un nombre considérable des règles qui recommandent ce qui est indispensable à sa réalisation. C'est ainsi qu'il recommande d'observer toujours les contrats par lesquels on s'est engagé, à ne jamais se montrer ingrat, à pardonner le passé à ceux qui témoignent du repentir, à ne favoriser personne lors d'un partage de biens, etc. etc.

Pour nous aider à compléter cette liste, Hobbes nous munit

d'une règle heuristique qui sert à découvrir et à éliminer les actions nuisibles à la paix. C'est la règle: «Ne faites pas à autrui ce que vous ne voudriez pas qu'on vous fit . . .» Un autre principe sert à délimiter l'application de la règle morale. Nous ne sommes tenus de la respecter, que lorsque notre partenaire fait de même.

La première question qui se pose quand il s'agit de systèmes de ce genre, c'est la question, si aucune forme de lutte n'est admissible. On sait qu'une société, composée de citoyens formant un troupeau homogène et docile, peut nous garantir la paix plus facilement qu'une société composée d'individus dont chacun tient à son indépendance, aussi bien quand il s'agit de ses goûts que de ses idées. Nul ne met en doute que les discussions, bien qu'elles engendrent parfois des antagonismes profonds, soient propices au développement de la culture. Allons nous éliminer tous les conflits à l'exception de ceux qui nous protègent de conflits encore plus redoutables? Si Hobbes admettait qu'il existât des conflits bienfaisants, ce qui paraît probable, il aurait dû nous munir d'un principe de sélection, qui nous aurait permis d'accepter certains conflits et de rejeter d'autres. Ce principe nous révélerait certainement un modèle de l'homme, dont la vie serait en harmonie avec celle de ses semblables, car la qualité de ceux qui doivent former un tout harmonieux ne peut être indifférente au moraliste.

En lisant Hobbes, on peut constater qu'en jugeant les hommes il se sert de deux mesures différentes et distingue les vertus des citoyens en tant que citoyens des vertus des citoyens en tant qu'hommes. La bienfaisance et la justice, constituent les principales vertus des hommes en tant que citoyens, tandis que des vertus telles que la magnanimité, le courage et la confiance sont honorables quand on considère les citoyens en tant qu'hommes, ces vertus étant une manifestation de force. Une fois encore un modèle de perfection vient compléter ici l'hygiène de la vie sociale.

II. *Morales à ambitions empiriques*

Les modèles de systèmes dont nous venons de parler et qui prennent la forme d'une pyramide en s'efforçant de déduire

toutes les règles de conduite d'un seul principe, ne sont, évidemment pas les seuls à se servir d'un schéma déductif. On connaît d'autres systèmes prenant pour point de départ une définition ou un axiome qui ne fixe aucun but ultime. Ces systèmes, de même que les précédents, exigent toujours des prémisses supplémentaires pour tenir compte de toutes les intuitions morales chères à l'auteur.

Nous allons discuter maintenant un modèle qui paraît se passer complètement de tout jugement de valeur et de tout élément normatif. C'est un système très en vogue à l'heure actuelle, surtout chez les écrivains anglo-saxons, mais on le retrouve déjà au XVIII^e siècle.

Ici, le moraliste commence par dresser une liste des besoins élémentaires des hommes. Ces besoins sont multiples et c'est le moraliste qui prend à tâche de trouver les meilleurs moyens permettant de les satisfaire. Ici, tout semble dicté par les faits. C'est en s'adressant aux faits, qu'on trouve les buts. C'est en s'adressant aux faits qu'on trouve les moyens. Le succès que remporte actuellement ce système a des raisons profondes. Ceux qui le proposent ne croient pas qu'on puisse parler de vérité en cas de jugements de valeur et de règles, ce qui fait douter de la possibilité de les rendre universels par la mise en relief de leur valeur logique. S'il est impossible de prouver la véracité de ces jugements et de ces règles, il reste toujours la possibilité de construire un système moral généralement approuvé en l'appuyant sur les besoins élémentaires communs à tous les êtres humains.

Ce modèle de système est soutenu actuellement aux États-Unis par des philosophes qui se considèrent opérationnistes. «L'opérationniste – lisons nous dans le livre de A. Rapoport publié en 1953 sous le titre de «Philosophie opérationniste» (Operational Philosophy) – pense qu'on peut construire un système de valeur général, basé sur les modes et les moyens de satisfaire les besoins généralement ressentis». Du moment que l'efficacité des moyens peut être prouvée par l'expérience, l'opérationniste a le droit de se placer à un point de vue super-culturel, car il peut critiquer l'efficacité des moyens à l'aide desquels diverses cultures réalisent différemment toujours les mêmes besoins. L'auteur cité

n'est pas le seul à soutenir ce schéma. Divers sociologues et ethnologues s'efforcent aujourd'hui de dresser une liste de ce qu'ils appellent besoins fondamentaux (*basic needs*), liste qui doit leur servir de point de départ pour la construction de la morale.

Si on commençait par demander, pourquoi les besoins élémentaires de l'homme doivent être satisfaits, on entendrait probablement la réponse: parce que l'homme souffre lorsque ses besoins élémentaires ne sont pas satisfaits. Le postulat nous engageant à satisfaire les besoins de l'homme – et constituant *un premier élément normatif*, non remarqué par les auteurs du système – paraît donc découler d'un postulat plus général qui nous recommande d'éliminer la souffrance en général, à moins qu'elle ne soit nécessaire pour éviter une souffrance encore plus grande.

Notons ce postulat et demandons ensuite, comment distinguer l'action approuvée par le moraliste parmi les nombreuses actions, que l'homme entreprend pour satisfaire ses besoins. Le paysan qui laboure son champ satisfait des besoins élémentaires, celui qui construit une maison les satisfait également. Toute activité est au service de tel ou tel autre besoin. Quand avons-nous affaire à une activité que nous jugeons vertueuse?

Maintenant, il nous faut nous arrêter un instant sur la notion des besoins élémentaires. C'est d'autant plus nécessaire que leur liste varie de beaucoup chez différents auteurs. Br. Malinowski qui attachait une très grande importance à dresser un inventaire des besoins communs à tous, mais réalisés dans différentes cultures de façons différentes, établissait une liste autre que celle que proposent ses collègues. Aussi longtemps qu'il traitait les besoins biologiques de l'individu, tels que le besoin de manger, de boire, de respirer, de dormir, le besoin de mouvement et de repos, de se protéger du froid, etc. la situation paraissait claire. Mais quand venaient s'ajouter aux besoins élémentaires ceux dont dépend non seulement la survie de l'individu mais aussi la survie de la société, sa liste se compliquait d'une façon inquiétante. L'auteur américain, A. Rapoport, que j'ai déjà cité, place au nombre des besoins élémentaires le besoin d'ordre qui se manifeste, entre autres, dans la science et dans l'art, un autre

besoin qu'il appelle besoin de l'extension du moi (*self extension*) et qui constitue en fait tout un groupe de besoins, comme, par exemple, le besoin de contacts sociaux, le besoin de faire partie d'un groupe (*the need to belong*), etc.

Même si nous ne tenons compte que des besoins biologiques de l'homme, tels que le besoin de manger, de boire, de respirer, de dormir, etc. la notion de l'élémentaire présente des difficultés. Le besoin de boire du vin d'une marque et d'une date définies n'est certainement pas élémentaire, comme ne l'est pas le besoin de manger du caviar. Si nous nous débarassons de cette objection en ne jugeant nécessaire que la satisfaction des besoins dont dépend notre survie, nous serons obligés d'admettre que les États-Unis et tous les pays de l'Europe où personne ne court le danger de mourir de faim ou du froid, n'ont pas de besoins élémentaires à satisfaire, et peuvent par conséquent se passer de tout système de morale.

On pourrait encore essayer un autre critère pour distinguer les besoins élémentaires d'avec ceux qui ne le sont pas, en ne traitant comme tels que les besoins qui sont toujours considérés plus importants en cas de conflit. Mais alors, la liste serait très individuelle. Ainsi, par exemple, certaines familles dans certaines sociétés se privent de nourriture pour garder leur position sociale; un amoureux se prive volontiers de dîner pour acheter un cadeau à sa bien-aimée, et celle-ci à son tour est toujours prête à jeuner pour s'acheter une robe qui l'avantagerait.

Laissons de côté pour un instant la notion d'élémentaire vu les difficultés auxquelles nous nous heurtons pour la définir et revenons aux besoins communs à tous. Ici, une question s'impose: sommes-nous obligés de respecter tout besoin suffisamment général? On peut admettre, que le bien, c'est ce qui satisfait un besoin – disait Shaftesbury – mais est-il bon d'avoir certains besoins? Nous n'avons rien contre une mouche qui se régale sur un fumier, mais si l'homme en faisait autant?

Supposons que le portrait de l'homme esquissé par Hobbes soit fidèle et que l'homme ait un besoin impérieux de se sentir supérieur. Allons-nous respecter ce besoin? Ici Bertrand Russell, qui, dans ses derniers ouvrages, suit une ligne analogue à celle:

de l'opérationniste américain mentionné, a une réponse toute prête. Nous ne devons pas respecter un besoin comme celui-ci – nous dira-t-il – car la satisfaction de besoins de ce genre entraîne la nécessité de ne pas satisfaire d'autres besoins inévitablement en conflit avec les précédents. Tandis que des désirs comme celui de dominer ne peuvent être rassasiés qu'en étouffant ceux des autres, Russell propose de ne considérer comme étant justes (right) que ceux qui sont en harmonie avec le plus grand nombre possible d'autres désirs. Selon lui – ce critère permet d'accorder à l'amour la priorité sur la haine, à la collaboration – la priorité sur la rivalité, à la paix – la priorité sur la guerre.

En effet, si on adopte le postulat exigeant de satisfaire le plus de désirs ou de besoins possibles, le même postulat nous permet de ne pas tenir compte de ceux qui tendent à étouffer les autres. Si ce principe de sélection était le seul qui fut admis, on pourrait s'en servir sans que la cohérence du système en souffrît. Mais il est difficile de s'imaginer un système de morale qui n'admettrait pas aussi d'autres principes de la sélection des besoins dignes d'être respectés, ainsi que des principes de la hiérarchisation de ces derniers, dans le cas où ils ne pourraient être satisfaits simultanément.

Les besoins d'ordre sexuel sont, sans aucun doute, tant universels, qu'élémentaires, néanmoins il paraît que toutes les cultures de notre globe leur imposent des freins. Dans presque toutes les cultures que nous connaissons, les relations sexuelles entre proches parents sont interdites bien que la parenté soit souvent définie d'une manière très différente. De nombreuses cultures défendent les relations sexuelles pré-nuptiales, d'autres les défendent pendant les périodes de deuil, qui sont parfois de longue durée, d'autres encore ne les admettent pas entre époux pendant la période de l'allaitement qui, dans certaines régions est souvent beaucoup plus longue qu'en Europe. Tous ces renoncements à la satisfaction des besoins d'ordre sexuel, sont imposés pour des raisons diverses et parfois aussi sans raison apparente. Très souvent, ils sont imposés pour réaliser des ambitions de perfection selon un modèle donné. Les Kwakiutls de l'île Vancouver, comme nous informe François Boas, qui a étudié si longtemps leur

coutumes, détruisent pendant les réceptions tout ce qu'ils ont de plus précieux, même l'huile laborieusement extraite des poissons. Ils le font pour réaliser un idéal de la personne. La tâche du moraliste qui construit une éthique visant la satisfaction des besoins communs à tous, serait certainement moins difficile, si le nombre de ces besoins était réduit à un minimum. Néanmoins le moraliste qui veut une société différenciée, composée d'individus ayant des besoins riches et variées accepte les difficultés dues à cette variété pour ne pas devoir renoncer à un idéal de la personne auquel il tient.

Nous venons de parler de la nécessité de compléter les systèmes de morale, visant la satisfaction des besoins universels et de les doter de principes de sélection permettant de distinguer les besoins dignes d'être respectés d'avec ceux qui ne méritent pas de l'être, ainsi que de principes qui décideraient de notre choix en cas de conflit. Parfois *la notion même du besoin* présuppose déjà une sélection. Il en est ainsi quand on oppose un besoin qu'on dit «réel» à ce qu'on nomme «caprice». Ici le «besoin» c'est un «besoin raisonnable, un besoin approuvé», alors que les caprices sont des besoins qui ne le sont pas.

Le dernier système examiné était considéré libre de tout élément normatif. Nous nous sommes efforcés de démontrer que ce n'était qu'une illusion. Aux éléments normatifs que nous avons déjà signalés, il y a lieu d'en ajouter encore un: notamment les règles qui permettent la sélection des moyens admissibles pour satisfaire les besoins universels, car il est hors de doute que le moraliste rejettera certains moyens, malgré leur efficacité. Une flatterie bien adressée peut être utile à un chômeur qui cherche du travail, mais le moraliste qui la juge déshonorante ne la recommandera certainement pas.

On peut se demander enfin, si la recherche des besoins élémentaires et universels n'est pas guidée par des motifs d'ordre moral, qu'il serait bon de dévoiler. Pourquoi ne devons nous tenir compte que de ceux qui sont universels? Est-ce uniquement pour donner une base solide aux principes moraux et pour garantir à ces principes la possibilité d'être généralement acceptés, ou est-ce aussi par ce que nous sommes tous égaux lorsqu'il

s'agit d'éprouver ces besoins. Mais si nous sommes guidés par un souci de justice, ce même souci nous oblige à trouver des règles qui nous permettraient de décider qui choisir, lorsque les mêmes besoins éprouvés par des personnes différentes ne peuvent être respectés en même temps. Il nous faut donc des règles de distribution. Pendant l'occupation allemande, les gens mouraient de faim dans le ghetto de Varsovie. Au moment où personne ne prévoyait encore l'extermination de ses habitants, les médecins du ghetto eurent un jour la chance de recevoir un paquet qui leur fut remis en cachette, malgré les murs qui les isolaient du monde. Ce paquet contenait des médicaments très précieux, des vitamines et des matières nutritives, destinés aux enfants. Hélas, il n'y en avait pas assez pour tous. Si on ne distribuait que de petites doses à tous les enfants, ces doses insuffisantes ne donneraient à aucun d'eux la chance de survivre. Peut-être était-il plus indiqué de choisir les enfants les moins chétifs et de leur administrer une dose plus grande qui leur permettrait de subsister? Fallait-il condamner tous les enfants en ne privilégiant aucun d'eux ou fallait-il en sauver une partie, en laissant mourir les autres? Le problème, *qui* nous devons privilégier, lorsque nous avons affaire à des besoins identiques ne pouvant être simultanément satisfaits, ainsi que le problème de notre droit de privilégier qui que se soit, ne se posent – heureusement – pas toujours d'une façon si cruelle, néanmoins ils doivent être résolus. Celui, qui dans les cas mentionnés, jugerait que le grand danger qu'entrevoyaient les habitants du ghetto leur imposait la fraternité et la solidarité dans le malheur jusqu'au bout, adopterait une règle qui, de nouveau, ne se laisserait pas déduire des prémisses du système.

Comme nous l'avons déjà dit, si nous demandons à ceux qui proposent ce système de morale, pourquoi ils veulent satisfaire les besoins élémentaires, et s'ils répondent que c'est pour éviter aux hommes la souffrance, nous avons le droit de traiter ce système comme un cas particulier des systèmes qui ont le bonheur en vue. La portée de ce système est toutefois plus limitée, car il ne s'occupe que des souffrances qui résultent des besoins élémentaires inassouvis. C'est au droit de satisfaire ces besoins

que pensaient les écrivains du XVIII^e siècle, lorsqu'ils parlaient des droits naturels. En Pologne, nous avons aussi au XVIII^e siècle des systèmes de morale qui commençaient par dresser une liste de droits, c'est-à-dire de besoins, dont la satisfaction pouvaient être réclamée. L'obligation de respecter ces droits entraînait autant de devoirs. Tous ces systèmes, attrayants par leur simplicité apparente, étaient incapables de résoudre les difficultés que nous venons de signaler.

Conclusions générales

Nous avons choisi pour notre analyse quelques systèmes auxquels la pensée humaine revenait plus d'une fois au cours des siècles. Les raisonnements qu'on y trouve étaient peu variés et se réduisaient à deux types principaux: tantôt on recommandait en général une certaine conduite et on montrait qu'un acte donné, appartenant justement à cette catégorie, constituait aussi un acte de mérite, tantôt, après avoir fixé un but à nos activités, on recommandait tout comportement nécessaire, suffisant, ou bien nécessaire et suffisant pour l'atteindre. Ces raisonnements ne sont pas les seuls possibles. Divers logiciens tentent actuellement, non sans succès, de créer une logique des normes. Le fait de ne pas pouvoir se servir dans ce domaine de la notion du vrai au sens d'Aristote ne nous empêche pas de parler de la vérité en tant que cohérence des normes dérivées et des normes acceptées en qualité d'axiomes. En suivant l'exemple des systèmes de logique nous pouvons exiger que ces axiomes soient indépendants et non contradictoires. La construction d'un système normatif exige toutefois un travail préliminaire, car il faut adapter nos notions d'implication, de négation, de contradiction, etc. à ce nouveau domaine. Bien que le choix entre différents systèmes qu'on pourrait nous proposer reste toujours libre et dépende avant tout de notre formation émotionnelle, il vaudrait mieux choisir *ceteris paribus* un tout cohérent et ordonné plutôt, qu'un ensemble décousu. En discutant le caractère scientifique des systèmes normatifs il est bon de se rappeler que le caractère scientifique d'un ouvrage dépend non d'une seule, mais de plusieurs qualités. Ce qui est clair a, du point de vue du théoricien,

plus de valeur que ce qui est obscur, les notions définies sont préférables à celles qui ne le sont pas, les jugements modestes l'emportent sur les généralisations hâtives, etc. Toutes ces qualités sont sujettes à une gradation et, si on ne peut faire d'un système normatif quelque chose de scientifique au sens exact de ce terme, on peut toutefois en faire quelque chose qui réalise au moins en partie les exigences que la logique impose aux systèmes.

Dans mon pays, les éducateurs attendent beaucoup d'un tel système. S'il s'agit de son effet sur la pratique, je suis beaucoup moins confiante, mais je ne doute pas que la théorie de la morale peut profiter des tâtonnements qui précédent et accompagnent ce travail de systématisation et c'est pourquoi je pense que cette tâche mérite que le moraliste lui consacre ses efforts.

DISCUSSIONS

SYMPOSIUM ON PHILOSOPHICAL ANALYSIS

Held at the First Scandinavian Philosophy Congress
in Hilleröd, Denmark, Summer 1959.

I

Philosophical Analysis and its Function. By *Justus Hart-
nack* (Aarhus Univ.).

To philosophize about philosophy has within later years come into discredit. The philosopher's task, so it is maintained, is not to philosophize about what it is to solve philosophical problems; it is to solve these problems. And this is of course true. A philosophy that is a philosophy about philosophy is neglecting the proper goal of philosophical activity: To solve philosophical problems. Not only much, but too much debate has been devoted to investigating the nature of philosophy and to show that the opponent was not really doing philosophy but doing something else.

But this does not mean that meta-philosophical investigations are taboo. Indeed, so far from being taboo, they are in demand. A greater understanding of the characteristic of the philosophical activity enhances the possibility of the progress of this very enterprise. I am here using the expression 'philosophical activity' and not the expression 'philosophical analysis'. This I do purposely. By philosophical analysis I do not understand a special method, a method among many, only incomparably better. By philosophical analysis I understand any kind of philosophical activity. This is not to narrow the concept 'philosophical activity' but is to widen the concept 'philosophical analysis'. And yet, this could hardly be quite right. Because it is possible to widen only such concepts whose meaning already is determined. And this could scarcely be said of the concept 'philosophical analysis'. There are some who have thought that the essence of philosophical analysis was to give definitions. Others have thought that definitions had nothing to do with philosophical analysis. There are some who have thought that philosophical analysis was the same as semantical investigations. Others have thought that this was a misunderstanding. And to say that philosophical analysis attempts to clarify that which is in need of clarification is trivial enough to be in need of no denial.

Let me, however, maintain the following negative but obviously true propositions. (1) Observations and experiments are irrelevant to the philosophical argument. (2) The philosophical argument, consequently, is not an argument about something empirically. (3) In the sense of 'to prove' in which mathematics proves its theorems, philosophy proves nothing. In philosophy there are no axioms, rules of deductions or theorems. Arguments for or against sense data, or for or against the possibility of absolute knowledge are not decided by any experimental technique, however refined, and there are no rules of deductions by help of which we can deduce either the possibility or the impossibility of absolute knowledge.

With respect to the purpose of philosophical analysis I shall answer briefly. The purpose is to throw light on philosophical problems. I use the expression 'to throw light on' and not the expression 'to solve'. And I do this purposely because it is often necessary for philosophical analysis to *discover* the philosophical problem – no philosophical problem can be solved that has not yet been discovered!

Not long ago all meaningful expressions were classified as descriptions and, consequently, as being either true or false. But to classify all meaningful expressions as descriptions creates philosophical problems. There are no difficulties in asserting, say, that all mailboxes in Denmark are red, but we cannot escape difficulties if, instead, I say that I know or that I believe that all mailboxes in Denmark are red. Or that it is true that all mailboxes in Denmark are red. For what am I describing? For reasons, unnecessary to outline here, nothing but absurdity results from classifying such expressions as descriptions. However, it is by now almost universally agreed that these expressions have a logic quite different from that of descriptions. To day it is understood that several kinds of expressions which do not belong to the category of descriptions have been wrongly categorized as descriptions. Thus the expression 'I know' may (in some contexts) be a *performatory* expression and the expression 'I believe' (in some contexts) a *parenthetical* expression. Nor can expressions like 'It is yours' and 'You did it' be categorized as descriptions. It is now seen that they ought to be categorized as *ascriptions*.

I have mentioned just a few cases of wrongly categorized expressions. Many more could be cited but enough have been given, I think, to bring home the following point. Some philosophical problems (if not all) are rooted in a misconception of the logical function of certain words and expressions. Some expressions are conceived of as belonging to one logical category while they really belong to another. Philosophical problems have arisen because expressions that really are either performatory or ascriptive or parenthetical or some other non-descriptive category have been wrongly classified as descriptive.

In other words: Philosophical analysis deals with the logical function of words and expressions. And since the philosophical problem arises because certain words or expressions are wrongly ascribed with certain logical properties and, consequently, with certain logical powers, it follows that the problem is solved by a correction of these logical misconceptions.

It is by this procedure that Hart has found and solved problems within philosophy of law, that Austin has thrown light on the concept of knowledge, that Urmson has helped us to see that verbs which we have been inclined to regard as descriptions of psychological states really are parenthetical, and that Urmson and Novell-Smith have made their valuable contributions toward a better understanding of the concept 'the good'.

It is often asserted that philosophy analyses ordinary language. The reason it is called 'ordinary language' is that it is about expressions we all employ, indeed have to employ, and to use independent of whatever language we speak and of what we speak about. We can neither avoid nor be without expressions such as 'I know that ...', 'It is yours', 'You did it' and numerous others. The use of the term 'analysis' is, in this context, rather misleading. What the philosopher does is to investigate (to analyze) the logical powers and the logical properties of such ordinary expressions as those just mentioned. It is such expressions whose logical status we are apt to overlook and it is, consequently, such expressions that are often the troublemakers.

It ought to be emphasized that there is no special method prescribed to find the logical powers of a certain expression. It is not by any deductive procedure, such as mathematics, that philosophers get their results and win their victories. If it were, there could be no arguments and counter-arguments in philosophy. In mathematics we do not argue, we prove; nor, as mentioned above, will the scientific or empirical methods be of any use. We have no laboratories in philosophy as we have laboratories in physics and psychology.

I shall now, by examples, illustrate (but not define) the nature of the philosophical argument.

Austin's argument for the performatory character of the expression 'I know' can be summarized thus:

To say that I know and to say that I am absolutely sure is not to say one and the same thing. But the difference is not that, in the case of knowing, I am *more* than absolutely sure, for there is nothing superior to being absolutely sure. To be absolutely sure is to be as sure as one can be. The difference is analogous to the difference between the expressions 'I promise' and 'I fully intend to'. Promising is not something superior, in the same scale as fully intending, for there is nothing in the scale superior to fully intending. By using the expression

'I promise' I entitle other people to act on it, something I do not do by using the expression 'I fully intend to'. And likewise with the difference between the expressions 'I know' and 'I am absolutely sure'. By using the expression 'I know' I give others my authority to act on it. I give others my word that things are as I say they are. I have staked my reputation on it. To say 'I know' is an act; it is to perform a rite or a ceremony. To say 'I swear' or 'I promise' is not to describe anything I am doing but is to do something. And, likewise, to say 'I know' is not to describe anything but is to do or to perform something.

Austin's argumentation (as here outlined) purports to show (but not to prove) the difference between 'I know' and 'I am absolutely sure' and to show (but not to prove) the logical analogy on the one side between 'I know' and 'I am absolutely sure' and on the other side between 'I promise' and 'I intend to'; and, finally, to show that neither 'I know' nor 'I promise' and 'I swear' are used to describe, but are performances.

Another example. Strawson argues against Russell's distinction between sentences that are true, sentences that are false, and sentences that are meaningless. His argument can be summarized in this way. The sentence, "The King of France is wise" is neither meaningless nor true nor false. Since everybody understands it, it is not meaningless. But it does not have any truth value. Whether it is true or false depends solely on its use. If the sentence was used to state a proposition in the sixteenth century it would have one truth value. If it was used in the seventeenth century it might be a proposition with the opposite truth value. Stated in the twentieth century it would be what Strawson calls a spurious application. If today anybody asserted: "The King of France is wise" he would not be saying anything *false* but something *pointless*. The proper answer would not be that it was not true; rather, it would be, as Peter Geach writes in *Analysis*: "Don't be silly! There isn't any King of France".

In other words: What Strawson has seen and Russell overlooked is the necessity of distinguishing between a sentence and its use. A sentence can be meaningful or meaningless but cannot have a truth value. A truth value is something that applies not to sentences but to sentences used to state propositions.

Strawson's argument is decisive, or at least important, for the discussion about meaning. It is also important for several other problems, metaphysical as well as epistemological – none of which I shall here examine, or even mention. The purpose of exhibiting these few cases from the philosophers' work-shop has been to show, partly that philosophy (or, if you wish, philosophical analysis) works with ordinary language (in the above mentioned sense of this expression), and partly

to illustrate (but not to define) the nature of the philosophical argument. It cannot be defined as no definition will include everything that is properly called philosophy and exclude what ought not to be so called. General formulae such as the one that the philosophical argument attempts to show the logical function of words or expressions are acceptable but uninformative. Among other general formulae is the one that to determine the logical function of an expression is to see what kinds of question it will be logically proper to ask with respect to that expression and what kinds of assumptions it will be logically proper to make in connection with that expression and what inference powers the expression has – all of which presuppose a general understanding of certain elementary logical features of the language.

In order to understand Austin's argument it is necessary to understand that it is logically impossible to say that one knows that all mail-boxes in Denmark are red but that possibly it is not so – but logically possible to say that one feels sure but that, nevertheless, one may be wrong. In order to understand Strawson's argument I must understand that the *same* sentence can be used to state *different* propositions, and that it has no meaning to ask for the truth value of a sentence but only to ask for the truth value of a sentence that is used.

The philosophical argument, or, if one prefers, philosophical analysis presupposes *some* understanding of the logical function of words and expressions. If such an understanding does not exist philosophical analysis is impossible, that is, no philosophical argument and, consequently, no philosophy would then exist.

II

Philosophical analysis; types and aims. By *Anders Wedberg* (Stockholm University).

I have had the advantage of reading professor Hartnack's contribution before writing down my own reflexions, and I shall take his paper as my starting-point. Hartnack considers "philosophical analysis" to be identical with "philosophical activity" in general, i.e. all activity aimed at throwing light on philosophical problems. What such activity means, he seeks to explain by examples from modern "Oxford philosophy" and by some general statements which, I think, essentially express an Oxonian point of view. With conscious simplification I would like to describe Hartnack's procedure thus: First he enlarges the concept "philosophical analysis" so as to make it equivalent to "philosophy" in general; then he restricts the latter concept so as to make it virtually synonymous with "modern Oxford philosophy". I prefer to use the ex-

pression "philosophical analysis" more conventionally, as a name for all those activities to which it is applied to-day according to a fairly general and acceptable usage. To me the Oxford school represents only one among several trends which it is convenient to classify as "analytical". Other persons and trends of importance to-day, whose work is largely of an "analytical" nature, are: Gottlob Frege, Bertrand Russell, G. E. Moore and the so-called Cambridge school, the Vienna circle and logical empiricism, Hägerström and Phalén and the Uppsala school in Sweden, Arne Naess and the Oslo school. According to this way of using "philosophical analysis", there is also much philosophy – good or bad – which is not analysis. If examples are requested, let me point at the many philosophical theories about the essence of matter from Democritus onward.

Such a usage implies that it is difficult to define "philosophical analysis" at all precisely. Nevertheless, analytical philosophers have something in common, even if this something is rather indefinite. Analysts are engaged in making more comprehensible or clarifying the meaning of terms, statements and arguments. The difference between making a text "more comprehensible" and "clarifying" it is, I think, quite obvious. When someone translates a Greek text into Swedish, he may make it more comprehensible for me. But the meaning of the text may still not be clear simply because its author wrote without clarity. When someone, in some way or other, rewrites and improves the text, he clarifies it. In analytical activity both processes are usually involved and more or less intimately intermingled.¹

In order to make meaning more comprehensible or to clarify it several alternative methods may be used. One is the method of *translation*. A translation may be of different kinds. Looking at the translations modern philosophy has had to offer, the following distinction is perhaps the most apparent. Some translations belong to the *same* natural language as the expression being translated or to that language enlarged by some special technical terms or symbols. (An example is afforded by the famous logico-empiricist definition: "A sentence is meaningful if and only if it is verifiable". Naturally, I do not here wish to express any opinion as to the correctness or suitability of this definition.) Other translations are to *another* language than that of the expression being translated. (An example is Russell's definition of "one" as " $\hat{\alpha}((\text{Ex}) \cdot \alpha = \iota' \times)$ ".) In modern philosophy the trends in-

¹ Instead of saying that the analyst studies "terms" or "statements" it is sometimes just as legitimate to say that he studies "notions" and "propositions", understood in a non-linguistic sense. If "fact" and "true proposition" are identified, it may sometimes be said that analysis deals with facts. However, like Hartnack I shall confine myself to the linguistic formulation.

fluenced by mathematical logic have endeavoured to create formalized languages into which larger or smaller portions of our natural languages can be translated. One may also make an expression more comprehensible or clarify it by *describing or fixing the rules* that in some respects determine its use. Such a description need not be equivalent to a translation. (A description of this kind may, for instance, be to the effect that the sign "plus" has a meaning in a certain language only when used between names of integers.) Wittgenstein and the modern Oxford philosophy to which Hartnack refers have shown a certain predilection for descriptions of this kind and less interest in translation. However, I shall not dwell further on the typology of analysis, which could fill a whole volume by itself and would quickly lead to subtleties and controversial problems. Instead I shall state seven theses that are by no means original but which, I hope, may still give rise to discussion. All seven express a common point of view.

In part I have formulated the theses in conscious opposition to what I take to be Hartnack's explicit or implicit opinions. In part they imply an opposition to opinions that I myself have previously held in a more or less conscious form. Anyone familiar with the most important trends in contemporary Scandinavian philosophy will easily recognize that the position expressed in my theses is akin to that of the so-called Oslo school. (Above all I have been influenced by my colleague in Stockholm, professor Harald Ofstad.)

Most contemporary philosophers agree, in principle, that it is impossible to discuss, with any hope of success, the classical problems and theories of philosophy without first submitting them to an analysis. Somehow or other there is something unsatisfactory in the way philosophers have formulated these problems, though opinions differ as to the exact way in which the formulations are defective. Arne Naess and the Oslo school point above all to *ambiguity*, and *lack of "preciseness"*. Hägerström and Phalén in Uppsala maintained that many of the formulations contain *absurd*, self-contradictory concepts. Moore thought that philosophical theories are often formally *irreconcilable with statements that, according to correct usage, express trivial truths of common sense*. The Vienna school and logical empiricism asserted that the formulations are *devoid of meaning*. Wittgenstein and the Oxford school seem to think that classical philosophy frequently employs words in a manner that either violates *the rules of "logical grammar"* or, at least, has not been envisaged by these rules. Of the various arguments used to substantiate the view that analysis is necessary to the discussion of classical problems, the least problematical seems to be that which points to their ambiguity. One may debate whether the Kantian problem: "Are there synthetic judgements *a priori*?" is "absurd" or "devoid of meaning" or a violation of "logical

grammar", but it is hardly possible to deny that the problem is lacking in preciseness, that it is reasonably open to various interpretations.

Many philosophical trends have not stopped at the thesis that philosophical analysis is necessary. They go on to maintain that analysis by itself is sufficient for the solution of the classical problems of philosophy. Contrary to this opinion I state my first thesis:

Thesis I. *Philosophical analysis – in the sense previously intimated – is not sufficient for the solution of the classical problems of philosophy.*

It is easy to see how, for example, the Uppsala school could be inclined to maintain the sufficiency of analysis or at least to acts as if analysis were sufficient. When once it has been established that the term "B" has an "absurd" meaning, the answer to the question: "Is A B?" is given. It is also easy to understand why, for instance, logical empiricism or Wittgenstein could think analysis sufficient. When we have seen that the sentence: "A is B" is devoid of "meaning" or that it violates "logical grammar", the question: "Is A B?" vanishes. If, however, a problem is ambiguous, it is *a priori* probable that divergent status will be given to the particular questions within the spectrum into which it is dissolved by analysis. Perhaps some of the questions invite the severe verdict that, for instance, the philosophers of Uppsala and Vienna and Wittgenstein used to pronounce; but others will probably be solved only by continued research into the reality with which they are concerned.

There is a common view that analysis is in some sense independent of empirical research. Thus Phalén once said that "judgements of conceptual analysis" form a kind of *a priori* judgement on a par with judgements of pure mathematics. Hartnack says that "observations and experiments" are irrelevant to philosophical analysis. Against such ideas I wish to assert:

Thesis II. *Philosophical analysis is not independent of empirical research.*

There are two main arguments for this thesis.

First. If analysis asserts something about how words are actually used, it asserts something about empirical facts. Observation of these facts, then, is evidently relevant for the evaluation of a proposed analysis. This holds irrespective of whether analysis takes the form of a translation within a given natural language, or of a translation from a natural to a formalized language, or of a description of linguistic rules.

Hartnack says that philosophical analysis is concerned with "expressions we all employ, indeed have to employ", expressions we have "to use independent of the language we happen to speak and of what we

speak about". He also says that "philosophical analysis presupposes (merely?) some understanding of the logical function of words and expressions". I get the impression that, in Hartnack's opinion, analysis presupposes merely that we actualize our implicitly existing understanding of our own language. Even if valuable analyses are sometimes arrived at by this method, this fact does not invalidate the thesis. Even if the philosophical analyst can develop an interesting analysis "aus der Tiefe des Bewusstseins", it should be possible to verify or falsify the analysis by observing how language is used. The situation is analogous to a situation common in analytical psychology. Freud is said to have conceived many of his theories (e.g. about "Fehlleistungen") by observing himself. This does not or should not imply that his theories are incapable of being tested by empirical research.

Second. Especially when analysis tries to substitute, for a given expression, another which, from some point of view, is better for the purpose of describing reality, analysis usually presupposes an empirical investigation of the reality to be described. For instance, many philosophers have studied our everyday psychological vocabulary. When reading such analyses, say, in Moore, Russell, Broad, Wittgenstein or Ryle, one finds that they are, so to speak, squinting all the time, with one eye on words, their meanings and uses, and one eye on psychological reality. And how could it be otherwise?

Some philosophers to-day draw a sharp dividing line between philosophical analysis and logical research. I quote Hartnack: "In the sense in which mathematics proves its theorems, philosophy proves nothing". Against this view I wish to maintain:

Thesis III. *Philosophical analysis is not independent of purely logical research.*

Of course there is some truth in what Hartnack says. I cannot "mathematically prove" that an expression has such and such a meaning or is used in such and such a manner. But at the same time I find Hartnack's words very misleading.

First. One way of clarifying the meaning of an expression is to draw consequences from sentences in which the expression occurs. Analysis by translation may, for instance, imply that two sentences are logically equivalent just in the sense that they are consequences of each other. The relation of logical consequence may often be so complex that it becomes apparent only after we have performed quite advanced logical operations.

Second. It is sometimes advantageous to perform an analysis in two steps, the first of which is empirical while the second is *a priori*, logical. By an empirical study of linguistic usage we first establish that an expression is used in a manner that satisfies certain general conditions

These conditions may then be formulated as "conditions of adequacy" for a definition of the expression; in other words, we formulate the purely logical problem of finding, within a prescribed class of expressions in an exact language, some expression which satisfies these conditions and then we prove that these conditions are in fact satisfied by the proposed *definiens* or *analysans*. A famous example of an analysis that may be considered as conforming to this pattern, is Tarski's definition of the concept of truth.

Third. The following is a central problem in the development of modern analytical philosophy. No simple formal logical rules hold for the natural languages. A classical example may illustrate what I mean. The inference: (i) *The apostles were Jews – Peter was an apostle – Hence, Peter was a Jew*, is logically valid. The inference: (ii) *The apostles were twelve – Peter was an apostle – Hence, Peter was twelve*, is not logically valid but simply nonsensical. It might be tempting to lay down the formal logical rule: *If "The A's are B" and "x is (an) A" are true, then "x is B" is true.* But (ii) is an exception to this rule in our natural language. Many reasons – which I shall not go into here – make it advantageous, in certain contexts, to have a language that admits of simple rules like the one just mentioned. Hence, attempts have been made to create such languages into which statements of our natural languages can be translated. A pioneer in this type of analysis was, as everybody knows, Gottlob Frege, the creator of the "Begriffsschrift", the inspirer of Russell and Wittgenstein. In analysis of this kind, linguistic research and purely logical research are inextricably mingled.

In certain philosophical circles there exists the view, usually not very clearly expressed, that philosophical analysis is something that can be conducted with a minimum of specific knowledge. The analyst merely requires the ability to analyse what he himself thinks (Phalén) or his own usage (Oxford) and the ability to read the classics of philosophy. On the contrary I assert:

Thesis IV. *Philosophical analysis can be successfully conducted only in intimate connection with empirical and/or logico-mathematical research.*

This thesis is merely a corollary to the preceding ones and needs no separate argument.

In many contemporary quarters it has been said that "philosophy is philosophical analysis" or that "philosophical analysis is the only reasonable task of philosophy". On the contrary I suggest:

Thesis V. *It is impossible to have analysis alone as one's philosophical programme, and all the reasons adduced for such a programme are bad reasons.*

The first part of this thesis is merely a reiteration of what I have already maintained. But let us look a little more closely at the counter-arguments.

One is an argument from the history of philosophy. It is said that, for the sake of historical continuity, philosophy ought to continue the study of the problems with which philosophers have traditionally been concerned. According to the Uppsala philosophers, Hågerström and Phalén, it was important to continue the study of the problems that, for instance, Kant and the German transcendental philosophers debated. According to the Oxford school, philosophers ought still to ponder over those questions that Berkeley, Hume, Russell, Moore and the young Ayer pondered over. Add to this the thesis that traditional problems can be solved by analysis alone, and the argument from the history of philosophy is completed, though it can be embellished by certain extrapolations. The Uppsala school maintained that the analysis of classical problems gives an insight into the "dialectic" of "common sense" (literally: "general consciousness", "allmänna medvetandet"), whereas the Oxford school asserts that such analysis reveals unknown facts about "the logical grammar of our everyday language".

I disagree with this argument for several reasons. (i) It seems "reactionary" to desire, at all costs, the continued study of classical problems, especially if one at the same time thinks these problems so deeply discredited. (ii) I doubt, as I have already stated, that the majority of classical problems can be solved merely by logical analysis. (iii) The anthropological hypothesis about "common sense", maintained by the Uppsala school, and the linguistic hypothesis about "common usage" entertained by the Oxford school, are unconfirmed and even rather unpalatable.

Another argument is one which I should like to call "the administrative argument". (It could also be called "the employment argument" or "the remainder argument".) It is said that established or as yet unborn special sciences divide between themselves the total available reality. All truths are scientific truths. What then is left for philosophers to do? What remains, one says, is the analysis or clarification of the language used in describing reality. In an extreme form, this point of view is presented in Wittgenstein's *Tractatus*.

Let us pass by the mysticism to which this argument leads. (Philosophical activity does not involve the assertion of any truths. Why then all these heavy volumes?) The argument rests upon a distinction between "philosophy" and "special sciences" that is entirely arbitrary.

It is sometimes thought that the results of philosophical analysis are incapable of being methodically tested. Thus, Hartnack insists that there is no "definite prescribed method by means of which the logical

properties of an expression are established". Neither mathematical nor empirical methods are, in his opinion, applicable in philosophical analysis.

There is, of course, a kernel of truth in Hartnack's statements. There is for the analyst no method which guarantees the finding of a desired analysis. With equal right, however, it can be said that there is no method which guarantees that a mathematician, a physicist or a biologist shall find the desired solution to some problem. On the whole, methods guaranteeing scientific results are scarce. But, in addition, Hartnack apparently means that there are no methods by which the correctness of a proposed philosophical analysis can be checked. I, on the contrary, prefer to maintain:

Thesis VI. *The impression that a philosophical analysis cannot be methodically checked is essentially due to the fact that the analysis is formulated much too imprecisely.*

Thesis VII. *A philosophical analysis ought to be formulated so as to make it testable in the same way as other scientific theories are testable.*

Obviously, the Oxford statement (mentioned by Hartnack) that the verb "know" is "performatory" cannot be methodically tested. The verb "know" has been and is being used by millions of English-speaking people with different philosophical outlooks, at different times and in different countries. Something similar holds for its approximate equivalents in other languages we may consider. Since the scope of the Oxford statements is indefinite, we do not know whose usage should be investigated. (Bolzano, in his *Wissenschaftslehre*, uses the word "Erkenntnis" ("knowledge") in the sense of "true judgement" ("wahres Urteil"). Can this be considered to be a possible counter instance?) In addition, the adjective "performatory" is explained in such vague terms that its application must remain problematical. The same holds for many analyses formulated by philosophers who are more or less oriented towards mathematical logic. Methods are just as important as results. The lack of method, which is so apparent in much modern philosophical analysis, is to my mind a serious shortcoming.

To mitigate the harshness of this judgement I should like to end with this – not too serious – reflexion. I come to a house with a thousand locked doors to which I have no keys. I go to a lock-smith who says: "I know that type of lock. They are probably manufactured at such and such a place, and they are probably constructed in such and such a way. Here are some keys; most of them will fit, I expect". I am disappointed and answer: "But which key fits which lock? I am not satisfied by such vague guesswork." And then I turn to another lock-smith who says: "I promise to study the matter methodically. I shall investigate the locks and give you labelled keys." He does, indeed,

proceed methodically, and I have to wait a long time. In the meantime, if I had accepted the offer from the first lock-smith, I should already have opened many of the doors. The situation in philosophical analysis is sometimes analogous. Russell, for instance, gave us an analysis of what he called "definite descriptions", but it is up to us to see where it fits (if only approximately).

III

Empirical Semantics in Oslo. By *Eivind Storheim* (Oslo university)¹.

In this short note I will give an outline of the semantical and philosophical program of the so-called "Oslo group" of philosophers, particularly as developed by Professor Arne Naess.

I will first explain the main concepts and methods employed in semantical investigations, and then briefly discuss their purpose and their relation to philosophy. Concerning conceptual framework and methodology I will mainly follow the exposition of Naess' main work, "Interpretation and Preciseness" (IP).

What strikes the key-note in this work is the trivial, but important truth that words and sentences do not have a definite meaning "in themselves", in isolation from the contexts in which they are used. The meaning of a given word or sentence varies with the situation in which it occurs and with the persons sending or receiving it. The analyst therefore starts with *occurrences* of a given word or sentence which is delimited by some spatial (textual) or time reference. The meaning of a word or sentence may be said to be a function of the persons using it and the situation in which it occurs. As people differ in level of knowledge and in their conceptions and also continually find themselves in new situations, it follows that different instances of one and the same word or sentence may vary in meaning. This, of course, does not preclude many words and sentences from having highly stable meanings. Surveying many occurrences of the same word, one may find that its different meanings do not differ widely or differ only within certain definite limits. So-called ordinary usage may be viewed as a statistically and conceptually important pattern of meaning, "meaning" being explicated in certain ways. In IP a conceptual apparatus and various methods for the investigation of meanings have been worked out, the main stress being on written communication.

Meanings are most easily studied indirectly by means of designations and formulations. In IP relations between these are of central im-

¹ Slightly revised version of a paper contributed to the first Scandinavian Congress of Philosophy, Hilleröd 1959.

portance. The relations concerned are all meaning-equivalence relations; the relata are termed *synonymic alternatives*. The chief term for meaning equivalence is "*synonymous with*". This term is made technically more precise in various directions or is given different operational definitions. Various tests for synonymy are devised. I will return to these later.

A designation or formulation which is synonymous with another designation or formulation is called an *interpretation* of the latter. If the relation between two designations or formulations T and U in relation to a definite person, or group of persons, is such that T allows interpretations which U does not allow (that is to say if there exist synonymic alternatives of T which are not also synonymic alternatives of U), and if all interpretations of U are interpretations of T, then U is said to be *more precise than* T. Such formulations as U are called *precizations*. In connection with the concept "more precise than" the concept of *definiteness of intention* is introduced. Let us imagine that a person P utters a formulation T. If some precizations of T are submitted to P it is possible to obtain a measure of P's definiteness of intention by investigating what precizations P can vouch for. If a *receiver* of a formulation interprets it in such a way that it transcends the intention of the *sender* of the formulation, the receiver is said to have made a *transintentional* interpretation. The same formulation may admit of many interpretations, or, if one wishes, may express many different propositions.

In analytical philosophy one often meets formulations of the following type: "By --- is meant the same as by ...", "--- and ... are synonymous", "to say --- is to say ...", etc. It is often anything but clear what is intended by such formulations. They can be termed *synonymy-sentences* and may be symbolized $\text{Syn}(ab)$ where "a" and "b" are designations or formulations and "Syn" designates some relation of synonymy. In order to investigate what might be intended by such synonymy-sentences, variables for *persons*, P, and *situations*, S, are introduced.

In terms of these concepts it is possible to interpret a synonymy sentence in different ways. If a person says: "a is synonymous with b", one can interpret it as follows: "All occurrences (instances) of the designation a are synonymous with all occurrences of the designation b for all persons P in all situations S". This interpretation will, however, very often be unpalatable. The following is more palatable: "Some occurrences of 'a' are synonymous with some occurrences of 'b' for some P in some S". This, of course, is a much weaker assertion. Sometimes a synonymy-sentence indicates for what persons in what situations it claims validity, as in the following sentence: "As interpreted by Tarski, 'equality' and 'logical identity' mean the same in arithmetic".

In this synonymy-sentence a specification is made as to the person (Tarski) and the situations (in arithmetic) for which the sentence is intended to be valid. These indications delimit the *field of validity* for the synonymy-sentence. Most indications of field of validity are either indefinite or more often completely lacking. In the above example nothing is said as to whether the synonymy-sentence is intended to cover all occurrences of "equality" and "logical identity" in Tarski's arithmetical writings or only a subclass of them. It seems most plausible, however, to interpret it as covering all occurrences of the mentioned designations.

A conceptually important distinction is that which is made between *meta-occurrences* and *use-occurrences* of designations. (These categories roughly correspond to the distinction between mention and use in Anglo-Saxon philosophy). If one talks about a designation, or if a designation occurs as definiendum in a nominal definition, the occurrence is a meta-occurrence. If a designation is used in a natural context the occurrence is a use-occurrence. It is important to remember these categories when one is presented with a synonymy-sentence. In the synonymy-sentence concerning Tarski there is nothing to indicate whether the sentence purports to cover meta-occurrences, use-occurrences or both. Tarski perhaps defines "logical identity" by means of a definiens formulation and declares later that he will use "equality" in the same sense. If this is the case that the synonymy-sentence is intended to cover, it might be valid for meta-occurrences only. It is very possible that Tarski, in spite of his decision concerning the use of designations "logical identity" and "equality", in fact uses them contrary to his decision. In that case a synonymy-sentence which is valid for meta-occurrences will not be valid for use-occurrences.

The crucial term in the conceptual apparatus has been "synonymous with". This characterizes the position of the Oslo group of empirical semantics as opposed to other contemporary schools of semantics or linguistic philosophy. The term "synonymous with" receives various explications or is made more technical in various ways. A family of concepts of synonymy is introduced, all of the concepts being linked to empirical procedures, though in differing degrees. One concept, Normative-synonymy (N-synonymy) refers to sameness of meaning in reference to explicitly given linguistic norms, especially as they occur in normative definitions. To test a synonymy-sentence interpreted as a hypothesis about the stipulated meaning of terms, the technique of *elementary analysis* is introduced. This technique consists in making a list of all definitional statements made by an author, together with their implications ("implications" in a sense rather different from "contextual implications"). On the basis of these a definition covering all or some of the definientia is established.

Another concept of synonymy, occurrence-synonymy, concerns use-occurrence of the term in question. A rather elaborated technique, *occurrence analysis*, has been constructed for this purpose. Also, various questionnaires have been devised in which reactions to linguistic terms are experimentally stimulated. Synonymy is here mostly explicated, in terms of truth-conditions.

What is the purpose of these concepts and methods and what relation do they have to philosophy?

Firstly, these conceptual refinements show the indefiniteness of many assertions about "the meaning" of terms or formulations. Emphasized is the fact the process of establishing meanings is very complicated and laborious. Hypotheses about the meanings or uses of terms are no more easy to confirm than are other hypotheses of science. But philosophers have often thought so. It is often the case in contemporary philosophy that assertions about synonymy of terms, declarations to the effect that this sentence is analytic or that one synthetic, and allocations of particular words to logical types or categories suffer, from the standpoint of empirical semantics, both from indefiniteness about what is asserted and from lacking or incomplete evidence as confirmation for the given hypothesis. As soon as a controversial hypothesis about the meaning or use of an expression is advanced, systematic empirical research is called for. Armchair methods are no substitutes in the case of disagreements. It is of course not the intention of the Oslo group that empirical research should be used in all cases. Armchair methods will often do. But by research, involving contact with one's subject matter, the fruitfulness and adequacy of one's conceptual scheme is most likely to be furthered.

Secondly, one can with reason say that an empirical study of the meanings and uses of language in actual communication provides a better position for attacking philosophical problems than the mere construction of explications on the basis of intuition. Using the research methods of empirical semantics one is often given insight into how "the" problem may be split up into several problems. For some of the problems solutions may be easy, for others not; and according to some precisations there may appear to be no problem at all. The Oslo group of empirical semanticists does not claim absoluteness or generality in its approach to analysis. It only emphasizes the importance of an empirical attitude and of the general requirements of an empirical methodology and is opposed to the rationalist view that philosophical analysis is more akin to deductive than inductive argumentation.

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